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# Deriving the Double Ratio $R = 1 - 6Re(\frac{\epsilon'}{\epsilon})$

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**NA48 Collaboration**

**Abstract**

The arguments leading to the expression of the ratios of weak decay amplitudes

$$\eta^{00} = \frac{< \pi^0\pi^0 \mid H_w \mid K_l^0 >}{< \pi^0\pi^0 \mid H_w \mid K_s^0 >} \tag{1}$$

and

$$\eta^{+-} = \frac{< \pi^+\pi^- \mid H_w \mid K_l^0 >}{< \pi^+\pi^- \mid H_w \mid K_s^0 >} \tag{2}$$

in terms of the CP violation parameters $\epsilon$ and $\epsilon'$ are reviewed. The NA48 experiment will directly measure, with greatest precision, the moduli of these amplitudes and deduce the double ratio of $2\pi$ decay rates

$$R = \frac{\mid \eta^{00} \mid^2}{\mid \eta^{+-} \mid^2}. \tag{3}$$

We then present a numerical calculation of Re$\frac{\epsilon'}{\epsilon}$ using the present world average values of $K_{s,l}^0$ mean lifetimes and $2\pi$ branching fractions and of the charge asymmetry $\delta$ of $K_{l3}$ decay as given by the Particle Data Group. In two additional calculations, a preliminary result for $\mid \eta^{+-} \mid$ by NA48 and a recent value for $\mid \eta^{+-} \mid$ obtained by CPLEAR, respectivley, are used instead.

# 1 Introduction

$K^0$ mesons are observed in nature in two modes as $K_s^0$ and $K_l^0$. These mesons are subject to weak interactions, strangeness is not conserved. They have definite and distinct masses $m_s$ and $m_l$ and mean life times $\tau_s$ and $\tau_l$. Their dominant non-leptonic decays into pions with corresponding branching fractions are: $K_s^0 \to \pi^+\pi^-$ (0.6861 $\pm$ 0.0028) and $\pi^0\pi^0$ (0.3139 $\pm$ 0.0028), and $K_l^0 \to \pi^+\pi^-\pi^0$ (0.1256 $\pm$ 0.0020) and $\pi^0\pi^0\pi^0$ (0.2112 $\pm$ 0.0027).

Theory [1,2] works with strangeness conserving states $K^0$,$\overline{K}^0$ which do not have definite lifetimes nor definite masses for weak decays. These states are not eigenstates of CP:

$$CP \mid K^0 >= - \mid \overline{K}^0 > \tag{4}$$

$$CP \mid \overline{K}^0 >= - \mid K^0 >, \tag{5}$$

but the linear combinations

$$\mid K_1^0 >= \frac{1}{\sqrt{2}}(\mid K^0 > + \mid \overline{K}^0 >); \qquad CP = -1 \tag{6}$$

$$\mid K_2^0 >= \frac{1}{\sqrt{2}}(\mid K^0 > - \mid \overline{K}^0 >); \qquad CP = +1 \tag{7}$$

are eigenstates of CP. If CP conservation holds,

$$\mid K_s^0 >=\mid K_2^0 > \tag{8}$$

$$\mid K_l^0 >=\mid K_1^0 > . \tag{9}$$

Since the CP eigenvalues of $2\pi$ and $3\pi$ states are $\eta_C^2\eta_P^2(-1)^l = +1$ and $\eta_C^3\eta_P^3(-1)^{l+L} = -1$ respectively for their orbital angular momenta $l, L = 0$, the CP conserving decays $K_s^0 \to 2\pi$ and $K_l^0 \to 3\pi$ are well realized, but the CP violating decay $K_l^0 \to 2\pi$ would be forbidden. By the observation of $K_l^0 \to \pi^+\pi^-$ and $\pi^0\pi^0$ with branching fractions $(2.067 \pm 0.035)\ 10^{-3}$ and $(0.936 \pm 0.020)\ 10^{-3}$ respectively, CP violation in the $K^0$ system has been established.

The primary source of occurrence of $K_l^0 \to 2\pi$ is a small modification, by the complex-valued CP violation parameter $\epsilon$, of the linear combinations (6) and (7) to

$$\mid K_s^0 >= \frac{1}{\sqrt{2(1+\mid \epsilon \mid^2)}}((1+\epsilon) \mid K^0 > +(1-\epsilon) \mid \overline{K^0} >) \tag{10}$$

$$\mid K_l^0 >= \frac{1}{\sqrt{2(1+\mid \epsilon \mid^2)}}((1+\epsilon) \mid K^0 > -(1-\epsilon) \mid \overline{K^0} >), \tag{11}$$

so that the observed $\mid K_{s,l}^0 >$ states acquire $\epsilon \mid K_{1,2}^0 >$ components which give rise to CP violating decays. This is called indirect CP violation or CP violation from state mixing.

The inequality of the CP violating $K_l^0 \to \pi^0\pi^0$ and $\pi^+\pi^-$ decay amplitudes (both relative to the respective CP conserving ones for $K_s^0$, i.e. $\eta^{00} \neq \eta^{+-}$) is known as direct CP violation or CP violation of the decay amplitude [4]. It is characterized by an additional complex-valued CP violation parameter $\epsilon'$. The principal interest of the NA48 experiment is the investigation of direct CP violation in the $K^0$ system [3]. While indirect CP violation is a

well known experimental fact, direct CP violation is still not quite established, and the aim of NA48 is to clear up this point.

In chapter 2 we concentrate on the indirect CP violation parameter $\epsilon$. The direct CP violation will be introduced and discussed in chapter 3. After that we express the double ratio R (3) through $\epsilon$ and $\epsilon'$ in chapter 4. In chapter 5 we calculate R using numbers from the Particle Data Compilation Tables [5] and compare it with two R values obtained from using a preliminary NA48 result from 1996 data and a CPLEAR result instead.

# 2 The parameter $\epsilon$ of indirect CP violation

The parameter $\epsilon$ in equations (10) and (11) is a small quantity. In case $\epsilon = 0$ we recover relations (6) and (7) which express the states $\mid K_2^0 >$ and $\mid K_1^0 >$ by $\mid K^0 >$ and $\mid \overline{K}^0 >$. Introducing a vector representation on the basis of orthogonal states $\mid K^0 >$ and $\mid \overline{K}^0 >$, one has

$$\mid K^0 >= \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \mid \overline{K}^0 >= \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

and

$$\mid K_1^0 >= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \qquad \mid K_2^0 >= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

$\mid K_1^0 >$ and $\mid K_2^0 >$ are orthogonal and rotated by $\phi = \pm 45^o$ respectively. In the case of CP violation $\mid K_s^0 >$ and $\mid K_l^0 >$ are given by

$$\mid K_s^0 >= \frac{1}{\sqrt{2(1+\mid \epsilon \mid^2)}} \begin{pmatrix} 1+\epsilon \\ 1-\epsilon \end{pmatrix}, \qquad \mid K_l^0 >= \frac{1}{\sqrt{2(1+\mid \epsilon \mid^2)}} \begin{pmatrix} 1+\epsilon \\ -1+\epsilon \end{pmatrix},$$

and they are, by virtue of $\epsilon \neq 0$, not quite orthogonal any more. This characterizes the effect of CP violation in that both $K_s^0$ and $K_l^0$ ($K_l^0$ with small probability of order $\epsilon$) have the common decay mode into $2\pi$.

The Hamiltonian H can be written in the $K^0$-$\overline{K}^0$ basis by a 2 x 2 matrix:

$$H = M - i\frac{\Gamma}{2} \tag{12}$$

where M and $\Gamma$ are Hermitian 2 x 2 matrices, called the mass and decay matrices. Because the neutral kaons are unstable against decay, H itself is not Hermitian. For brevity we use the following short notation:

$$H_{mn} \quad = \quad < n \mid H \mid m > \quad = \quad M_{mn} - i\frac{\Gamma_{mn}}{2} \tag{13}$$

with m and n = 1,2 or $K^0$,$\overline{K}^0$. The diagonalisation of the eigen value equation

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \mid \psi > = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \mid \psi >$$

yields the two eigenvalues $\lambda_{s,l} = m_{s,l} - i\gamma_{s,l}/2$ and the two eigenstates $\mid \psi >=\mid K^0_{s,l} >$ in the $K^0$,$\overline{K}^0$-basis. Putting the determinant of $H_{mn} - \lambda\delta_{mn}$ to zero, solving

$$(H_{11} - \lambda)(H_{22} - \lambda) - H_{12}H_{21} = 0$$

for $\lambda$ and taking into account CPT-invariance ($H_{22} = H_{11}$), we obtain

$$\lambda_s = H_{11} + \sqrt{H_{12}H_{21}} \tag{14}$$

and

$$\lambda_l = H_{11} - \sqrt{H_{12}H_{21}}. \tag{15}$$

The eigenvector $\mid \psi >=\mid K^0_s >$ is obtained by solving the eigenvalue equation for $\lambda = \lambda_s$:

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} H_{11} + \sqrt{H_{12}H_{21}} & 0 \\ 0 & H_{11} + \sqrt{H_{12}H_{21}} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

with the result

$$a = \frac{\sqrt{H_{12}}}{\sqrt{H_{21}}} b. \tag{16}$$

Comparing this to (10) we recognize

$$a = \frac{1+\epsilon}{\sqrt{2(1+\mid \epsilon \mid^2)}}, \qquad b = \frac{1-\epsilon}{\sqrt{2(1+\mid \epsilon \mid^2)}}.$$

Solving these equations for $\epsilon$ and taking into account (16) we get

$$\epsilon = \frac{\sqrt{H_{12}} - \sqrt{H_{21}}}{\sqrt{H_{12}} + \sqrt{H_{21}}}.$$

By a simple transformation one obtains the following expression:

$$\epsilon = \frac{H_{12} - H_{21}}{4\sqrt{H_{12}H_{21}} + (\sqrt{H_{12}} - \sqrt{H_{21}})^2}.$$

Neglecting the difference $(\sqrt{H_{12}} - \sqrt{H_{21}})^2$ gives

$$\epsilon \approx \frac{H_{12} - H_{21}}{4\sqrt{H_{12}H_{21}}}, \tag{17}$$

which, by inserting the complex-valued matrix-elements (13) into $H_{12}$ and $H_{21}$ and disregarding $Im\Gamma_{12}$ due to $\mid Im\Gamma_{12} \mid \ll \mid ImM_{12} \mid$, yields

$$\epsilon \approx -i\frac{ImM_{12}}{2ReM_{12}}. \tag{18}$$

The expression

$$\mid \epsilon \mid \approx \frac{ImM_{12}}{2ReM_{12}}, \tag{19}$$

with $M_{12}$ operating on the $K^0$-$\overline{K}^0$ basis, is used for calculating $\epsilon$ with the Standard Model.

Transforming (17) with (14) and (15) we obtain

$$\epsilon \approx \frac{ImM_{12}}{i(\lambda_l - \lambda_s)}.$$

Taking into account

$$\lambda_l - \lambda_s = m_l - m_s - \frac{i}{2}(\gamma_l - \gamma_s) = \Delta m - \frac{i}{2}\Delta\gamma$$

we get for $\epsilon$

$$\epsilon \approx \frac{ImM_{12}}{\frac{1}{2}\Delta\gamma - i\Delta m}. \tag{20}$$

# 3 $\epsilon'$, the parameter of direct CP violation

We consider the transition amplitudes of $K_s^0 \to \pi\pi$ , $K_l^0 \to \pi\pi$ under the assumption that CPT invariance is valid. Pions are bosons therefore the total wavefunction must be symmetric with respect to particle interchange. The isospin of a pion is I=1 and therefore the final state can have only I = 0 or 2, $I_3 = 0$. This results from Clebsch - Gordan coefficients for I($\pi\pi$) = 0,1,2 [5] :
the symmetric wavefunctions are

$$\begin{array}{rcccl} & \mid 11> & \mid 1-1> & : \sqrt{\frac{1}{3}} & \pi^+\pi^- \\ \mid 00> \to & \mid 10> & \mid 10> & : -\sqrt{\frac{1}{3}} & \pi^0\pi^0 \\ & \mid 1-1> & \mid 11> & : \sqrt{\frac{1}{3}} & \pi^-\pi^+ \end{array}$$

$$\begin{array}{rcccl} & \mid 11> & \mid 1-1> & : \sqrt{\frac{1}{6}} & \pi^+\pi^- \\ \mid 20> \to & \mid 10> & \mid 10> & : \sqrt{\frac{2}{3}} & \pi^0\pi^0 \\ & \mid 1-1> & \mid 11> & : \sqrt{\frac{1}{6}} & \pi^-\pi^+ \end{array}$$

and the asymmetric wavefunctions

$$\begin{array}{rcccl} & \mid 11> & \mid 1-1> & : \sqrt{\frac{1}{2}} & \pi^+\pi^- \\ \mid 10> \to & \mid 10> & \mid 10> & : 0 & \pi^0\pi^0 \\ & \mid 1-1> & \mid 11> & : -\sqrt{\frac{1}{2}} & \pi^-\pi^+ \end{array}$$

The matrixelements in the definitions of $\eta^{00}$ and $\eta^{+-}$ (3) and (1) are now developed according to the symmetric Isospin Clebsch-Gordan coefficients.

$$\eta^{+-} = \frac{\frac{1}{\sqrt{3}} < \pi\pi, I=2 \mid H_w \mid K_l^0 > + \frac{\sqrt{2}}{\sqrt{3}} < \pi\pi, I=0 \mid H_w \mid K_l^0 >}{\frac{1}{\sqrt{3}} < \pi\pi, I=2 \mid H_w \mid K_s^0 > + \frac{\sqrt{2}}{\sqrt{3}} < \pi\pi, I=0 \mid H_w \mid K_s^0 >}$$

Dividing by $\frac{\sqrt{2}}{\sqrt{3}} < \pi\pi, I=0 \mid H_w \mid K_s^0 >$ and with the definitions of $\epsilon$ and $\omega$

$$\epsilon = \frac{< \pi\pi, I=0 \mid H_w \mid K_l^0 >}{< \pi\pi, I=0 \mid H_w \mid K_s^0 >} \tag{21}$$

$$\omega = \frac{< \pi\pi, I=2 \mid H_w \mid K_s^0 >}{< \pi\pi, I=0 \mid H_w \mid K_s^0 >} \tag{22}$$

we get

$$\eta^{+-} = \frac{\frac{1}{\sqrt{2}} \frac{<\pi\pi, I=2 | H_w | K_l^0>}{<\pi\pi, I=0 | H_w | K_s^0>} + \epsilon}{1 + \frac{1}{\sqrt{2}}\omega} \tag{23}$$

Adding and subtracting at the same time $\frac{\omega\epsilon}{\sqrt{2}}$ to (23) and by the definition of $\epsilon^{'}$

$$\epsilon^{'} = \frac{\epsilon}{\sqrt{2}}\left(\frac{< \pi\pi, I=2 \mid H_w \mid K_l^0 >}{< \pi\pi, I=0 \mid H_w \mid K_l^0 >} - \omega\right) \tag{24}$$

we get

$$\eta^{+-} = \epsilon + \frac{\epsilon^{'}}{1 + \frac{\omega}{\sqrt{2}}} \tag{25}$$

and similarily

$$\eta^{00} = \epsilon - \frac{2\epsilon^{'}}{1 - \sqrt{2}\omega} \tag{26}$$

Neglecting $\omega$ ($= 0.045$ [5]) we obtain the well known approximations

$$\eta^{+-} \approx \epsilon + \epsilon^{'}, \qquad \eta^{00} \approx \epsilon - 2\epsilon^{'} \tag{27}$$

Solving these two equations for $\epsilon$ and $\epsilon^{'}$ gives

$$\epsilon^{'} = \frac{1}{3}(\eta^{+-} - \eta^{00})(1 - \frac{\omega}{\sqrt{2}} - \omega^2), \qquad \epsilon = \frac{1}{3}(2\eta^{+-} + \eta^{00} + \sqrt{2}\omega(\eta^{+-} - \eta^{00})) \tag{28}$$

$\epsilon^{'}$ and $\epsilon$ are now expressed by measureable quantities $\eta^{00}$, $\eta^{+-}$ and $\omega$.

# 4 The Ratio R

Using ( 27) (3) can be expressed by $\epsilon$ and $\epsilon^{'}$.

$$R \approx \frac{(\epsilon - 2\epsilon^{'})(\epsilon - 2\epsilon^{'})^*}{(\epsilon + \epsilon^{'})(\epsilon + \epsilon^{'})^*}$$

and after some calculation and neglecting terms of the order of $\epsilon^{'2}$ we get the following expression

$$R \approx \frac{1 - \frac{2}{|\epsilon|^2} 2Re\epsilon^{'}\epsilon^{*}}{1 + \frac{1}{|\epsilon|^2} 2Re\epsilon^{'}\epsilon^{*}}$$

R can be developed according to $\frac{1}{1+x}$ for $\mid x \mid < 1$. And we receive

$$R \approx 1 - 6\frac{Re\epsilon^{'}\epsilon^{*}}{\mid \epsilon \mid^2} = 1 - 6Re\frac{\epsilon^{'}}{\epsilon}. \tag{29}$$

R is now expressed by $\frac{\epsilon^{'}}{\epsilon}$ which is directly determinable by experiment by (27).

# 5 Calculating R from Decay Rates, Lifetimes and 2π branching ratios

From (27) we get

$$\mid \eta^{+-} \mid^2 \approx \mid \epsilon \mid^2 (1 + 2Re\frac{\epsilon^{'}}{\epsilon}) \tag{30}$$

$$\gamma(K_l \to all) = \frac{\hbar}{\tau_l}$$

$$\gamma(K_l \to \pi^+\pi^-) = \frac{\hbar}{\tau_l} B(K_l \to \pi^+\pi^-)$$

$$\gamma(K_s \to all) = \frac{\hbar}{\tau_s}$$

$$\gamma(K_s \to \pi^+\pi^-) = \frac{\hbar}{\tau_s} B(K_s \to \pi^+\pi^-)$$

$$\tau_s = (0.8927 \pm 0.0009)\ 10^{-10} s$$

$$\tau_l = (5.17 \pm 0.04)\ 10^{-8} s$$

$$B(K_s \to \pi^+\pi^-) = (0.6861 \pm 0.0028)$$

$$B(K_l \to \pi^+\pi^-) = (2.067 \pm 0.035)\ 10^{-3}$$

$$\frac{\gamma(K_l \to \pi^+\pi^-)}{\gamma(K_s \to \pi^+\pi^-)} = (0.5202 \pm 0.00993)\ 10^{-5}$$

In order to determine $Re\frac{\epsilon^{'}}{\epsilon}$ form (30) we need to get $\mid \epsilon \mid^2$. [5] gives a value for the the charge asymmetry

$$\delta \approx 2Re\epsilon = (0.327 \pm 0.012)\%$$

$$Re\epsilon = (0.1635 \pm 0.006)\ 10^{-2}$$

From $Re\epsilon$ we find

$$| \epsilon |^2 = (Re\epsilon)^2 * (1 + \tan^2 \Phi)$$

$$\tan(\Phi) = \frac{\Delta m}{0.5\Delta\gamma}$$

$$\Delta m = (0.5304 \pm 0.0014)\ 10^{10}\hbar s^{-1}$$

$$\Delta\gamma = \hbar(\tau_l^{-1} - \tau_s^{-1}) = -(1.1183 \pm 0.00113)\ 10^{10}\hbar$$

Knowing all these values we can determine

$$Re\frac{\epsilon'}{\epsilon} = 0.01213 \pm 0.0284$$

For comparison we quote here the result evaluated by I.Mikulec:

$$Re\frac{\epsilon'}{\epsilon} = (1.2 \pm 1.2)\ 10^{-2}$$

and also a further NA48 result [6]:

$$Re\frac{\epsilon'}{\epsilon} = (-0.1603 \pm 3.014)\ 10^{-2}$$

and also a preliminary CPLEAR value [7]:

$$Re\frac{\epsilon'}{\epsilon} = (0.308 \pm 0.332)\ 10^{-1}$$

# 6 Summary

We have reviewed that theoretical quantities of CP violation can be expressed by experimental measurements. These theoretical quantities are

- $\epsilon'$ the direct CP violation parameter
- $\epsilon$ the indirect CP violation parameter
- the double ratio R
- $\omega$, $\eta^{00}$, $\eta^{+-}$.

They can be expressed by the decay amplidudes $< \pi\pi \mid H_w \mid K^0_{s,l} >$. To develop these expressions no physical model was used. With the help of Particle Data Booklet Data, NA48 Data and CPLEAR Data we have obtained estimates of $\frac{\epsilon'}{\epsilon}$ of low precisions. These estimates show that a high precision experiment is very important to pin down the real value of $\frac{\epsilon'}{\epsilon}$.

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