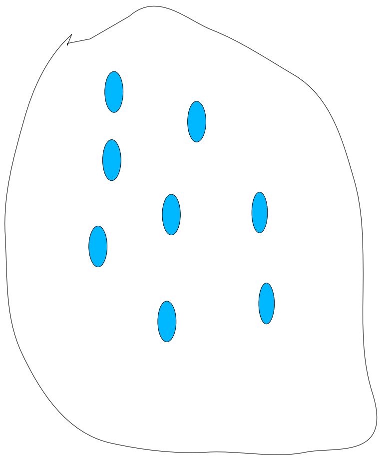
Corpi rigidi

quantita' di moto



$$\vec{Q}=\sum_{i=0}^{N} q_i=\sum_{i=0}^{N} m_i \vec{v}_i = M\, v_b$$

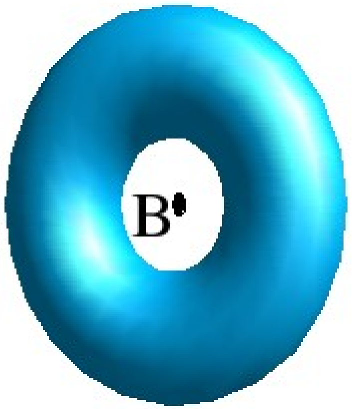
$$\vec{r}_B=\frac{1}{M}\sum_{i=0}^{N} m_i \vec{r}_i$$

$$\vec{Q}_B=\int_V \vec{v}\,\rho\, dV = M\, v_b$$

$$\vec{r}_B=\frac{1}{M}\int_V \vec{r}\,\rho\, dV$$

Corpi rigidi

Baricentro



$$\vec{r}_B = \frac{1}{M}\int_M \vec{r}\, dm = \frac{1}{M}\int_V \vec{r}\, \rho\, dV$$

$$x_B = \frac{1}{M}\int_V x\, \rho\, dV$$

$$y_B = \frac{1}{M}\int_V y\, \rho\, dV$$

$$z_B = \frac{1}{M}\int_V z\, \rho\, dV$$

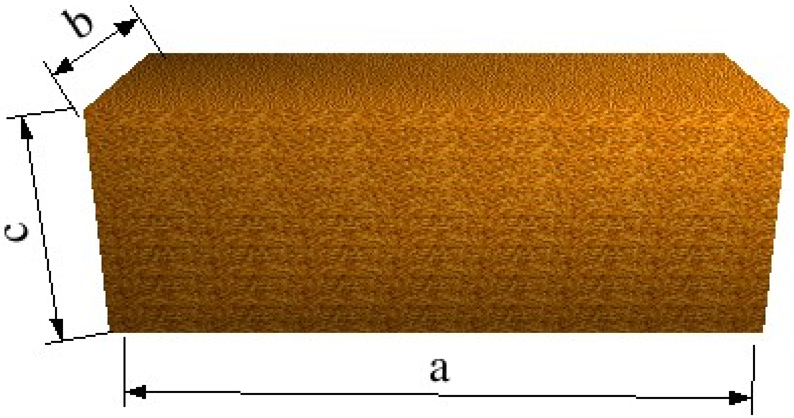
se ρ costante

$$\vec{r}_B = \frac{1}{V}\int_V \vec{r}\, dV$$

Corpi rigidi

Baricentro

Mattone

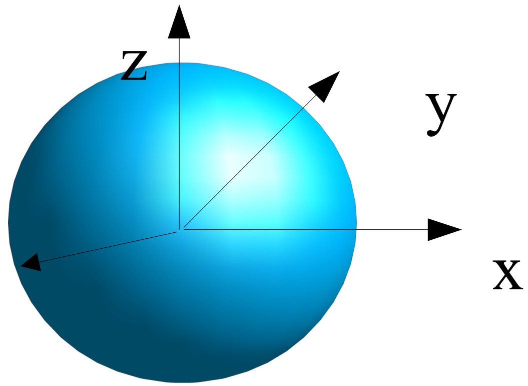


$$x_B = \frac{1}{M}\int_V x\,dm = \frac{1}{\rho V}\int_V x\,\rho\,dV = \frac{1}{V}\int_V x\,dV = \frac{1}{V}\iiint x\,dxdydz$$

$$\frac{1}{V}\iiint x\,dxdydz = \frac{1}{V}\int_0^a x\,dx\int_0^b dy\int_0^c dz = \frac{1}{V}\frac{a^2}{2}b\,c = \frac{a}{2}$$

e così via

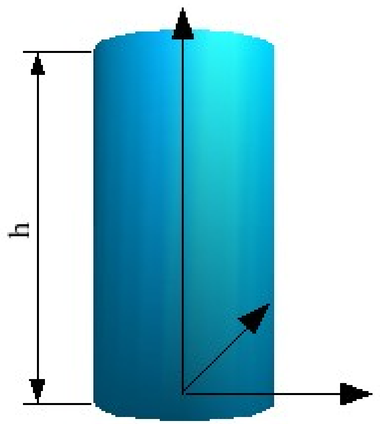
Sfera piena



$$x_B = \frac{1}{V}\int_V x\,dV = \frac{1}{V}\iiint r\cos\phi\sin\theta \;\; r^2\,dr\,d\phi\,d\cos\theta$$

$$x_B = \frac{1}{V}\int_0^R r^3\,dr \int_0^{2\pi}\cos\phi\,d\phi \int_{-1}^{1}\sin\theta\,d\cos\theta = 0$$

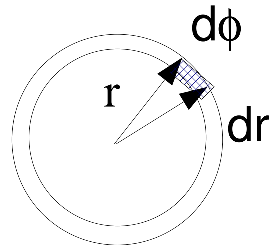
Cilindro pieno



$$x_B = \frac{1}{V}\int_V x\,dV = \frac{1}{V}\iiint r\cos\phi \;\; r\,dr\,d\phi\,dz = 0$$

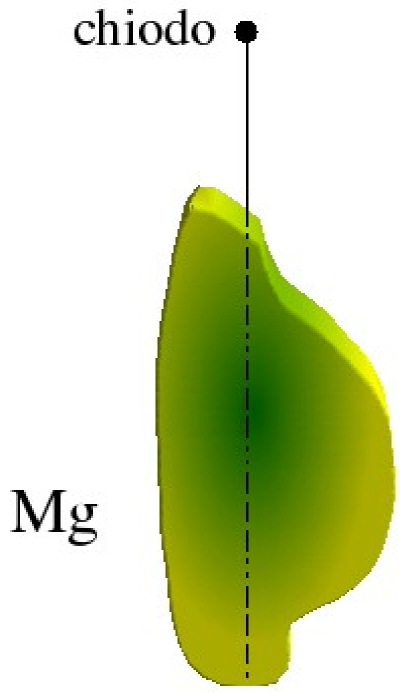
$$z_B = \frac{1}{V}\int_V z\,dV = \frac{1}{V}\iiint z\,r\,dr\,d\phi\,dz = \frac{1}{V}\int_0^R r\,dr\int_0^{2\pi} d\phi\int_0^h z\,dz$$

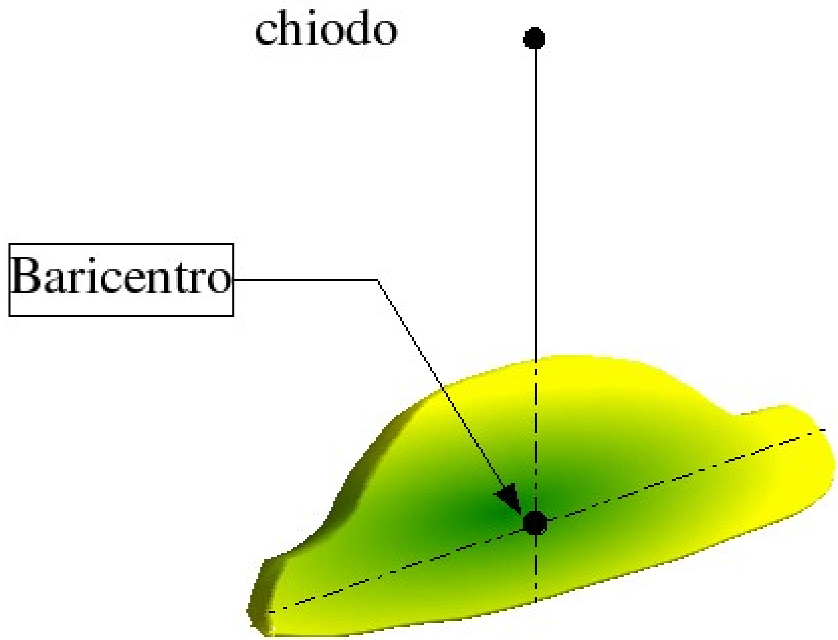
$$z_B = \frac{2\pi}{V}\frac{R^2}{2}\frac{h^2}{2} = \frac{h}{2}$$



Corpi rigidi

Ricerca del baricentro





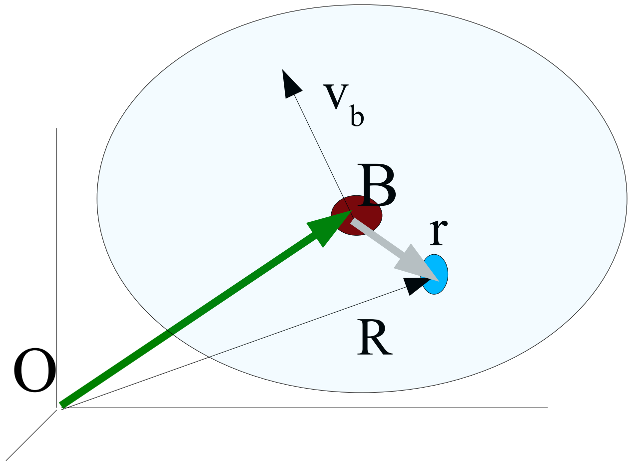
Corpi rigidi

momento angolare del corpo rigido

$$\vec{L} = \int_M \vec{R} \wedge \vec{V}\, dm = \int_M (\vec{R}_B + \vec{r}) \wedge (\vec{V}_B + \vec{r})\, dm = L_B + L_r$$

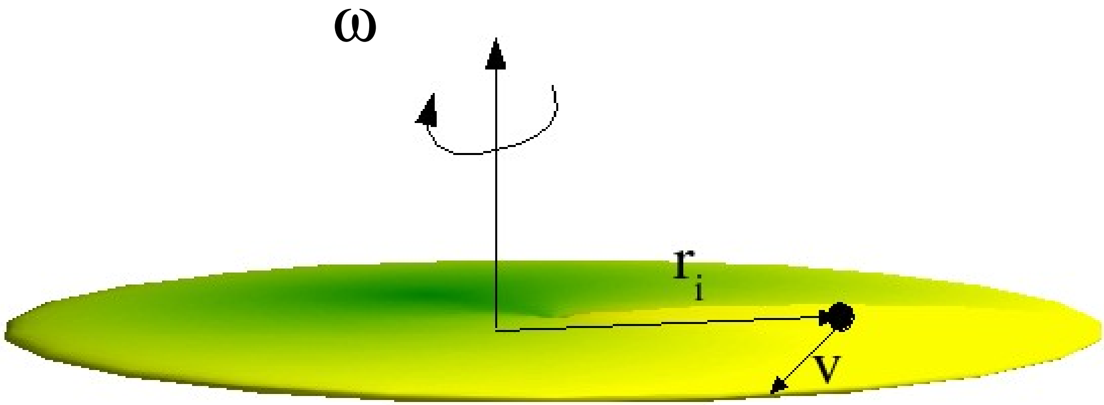
$$\vec{R} = \vec{R}_B + \vec{r}$$

$$\vec{V} = \vec{V}_B + \vec{v}$$



Corpi rigidi

momento angolare



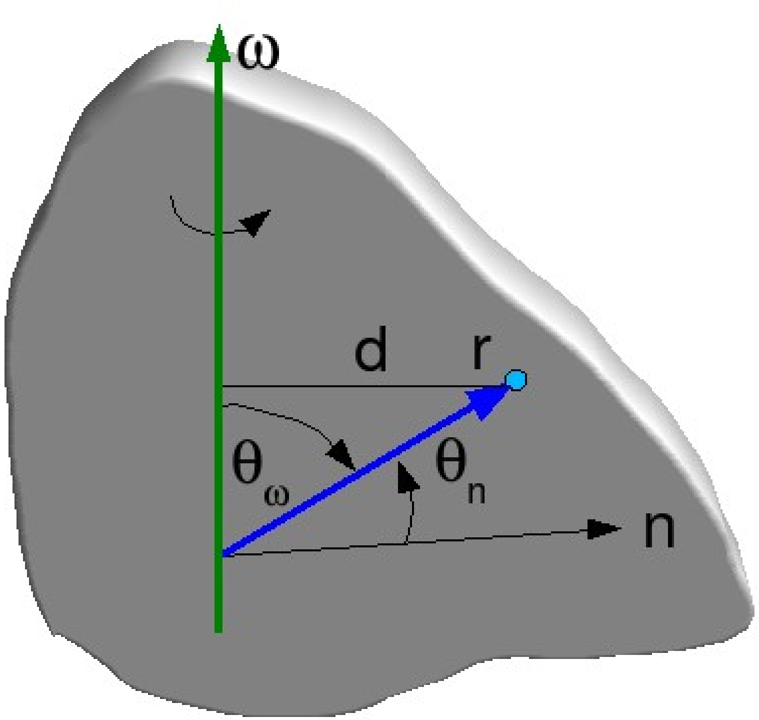
$$\vec{v}_i = \vec{\omega} \wedge \vec{r}_i$$

$$\vec{L}_r = \int_M \vec{r} \wedge \vec{\omega} \wedge \vec{r}\, dm = \int_M \left(r^2 \vec{\omega} - \vec{r} \cdot \vec{\omega}\, \vec{r} \right) dm$$

???

Corpi rigidi
momento angolare

$$\vec{L}_r \cdot \hat{\omega} = \int_M (r^2 \vec{\omega} - \vec{r} \cdot \vec{\omega}\, \vec{r}) \cdot \hat{\omega}\, dm = \omega \int_M (r^2 - (\vec{r} \cdot \hat{\omega})^2) \cdot \hat{\omega}\, dm$$



$$L_\omega = \omega \int_M d^2\, dm = I_\omega \omega$$

$$\vec{L}_r \cdot \widehat{\omega}_n = L_n = -\omega \int_M r^2 \cos\theta_\omega \cos\theta_n\, dm$$

non nullo!

$$\vec{L} \neq \vec{\omega}$$

Corpi rigidi

r = r ≡ (x,y,z), e **ω** ≡ $(\omega_x, \omega_y, \omega_z)$

Sviluppando

$$\vec{L}_r = \int_V \begin{vmatrix} r^2 - x^2 & -xy & -xz \\ -yx & r^2 - y^2 & -y\,z \\ -zx & -zy & r^2 - z^2 \end{vmatrix} dm \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{vmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{vmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

$$\vec{L}_r = I\,\vec{\omega}$$

$$con \quad I_{ij} = \int_M (r^2 \delta_{ij} - x_i x_j)\, dm$$

$con \quad \delta_{ij} = 1$ *se i=j oppure 0 se i diversa da j*

Corpi rigidi

tensore di inerzia diagonale

$$\vec{L}_r = \begin{vmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{vmatrix} \begin{pmatrix} 0 \\ 0 \\ \omega_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ I_z \omega_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ L_z \end{pmatrix}$$

$\vec{L} \parallel \vec{\omega}$

$$I_z = \int_M (r^2 - z^2)\, dm$$

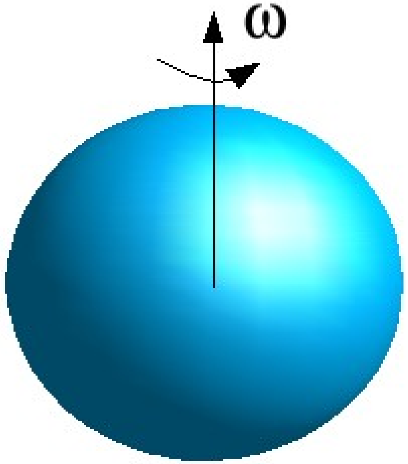
$$M_z = \frac{d\, I_z \omega_z}{dt}$$

Corpi rigidi

Momento di inerzia assiale - Sfera Piena

$$I_z = \int_V (r^2 - z^2)\rho\, dV = \int_V r^2 \sin\theta^2 \rho\, r^2\, dr\, d\phi\, d\cos\theta$$

$$I_z = 2\pi \int_0^R r^4 \rho\, dr \int_{-1}^{1} (1 - \cos\theta^2)\, d\cos\theta = \frac{2}{5} M R^2$$

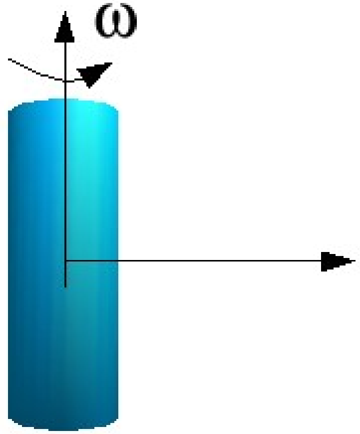


Corpi rigidi

Momento di inerzia assiale - Cilindro pieno

$$I_z = \int_V r^2 \rho \, dV = \int_V r^2 \rho \, r \, dr \, d\phi \, dz$$

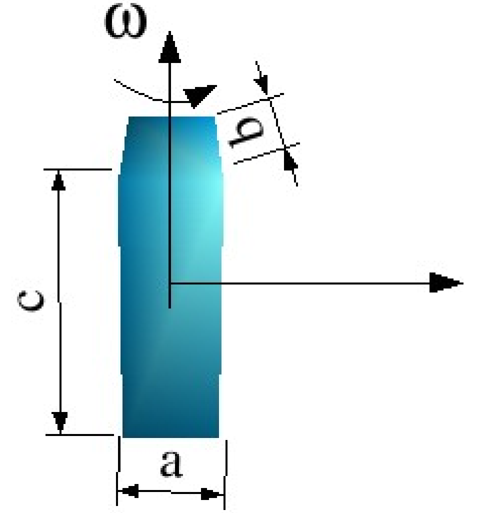
$$I_z = 2\pi\rho \iint r^3 \, dr \, dz = \frac{1}{2} M R^2$$

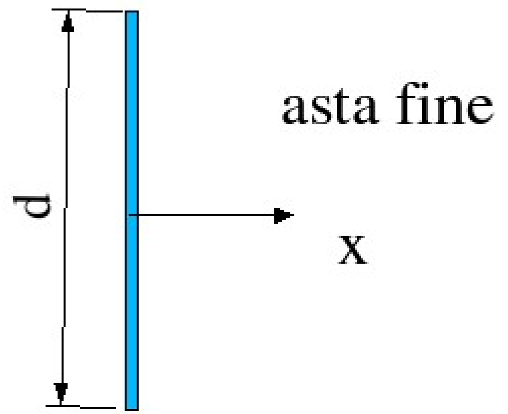


Corpi rigidi

Momento di inerzia assiale - Cilindro pieno

$$I_z = \int_V (x^2 + y^2)\rho\, dV = \rho \int_{-a/2}^{a/2} x^2 dx \int_{-b/2}^{b/2} dy \int_{-c/2}^{c/2} dz + \rho \int_{-a/2}^{a/2} \rho\, dx \int_{-b/2}^{b/2} y^2 dy \int_{-c/2}^{c/2} dz$$





$$I_z = \frac{1}{12} M (a^2 + b^2)$$

$$I_x = \frac{1}{12} M d^2$$