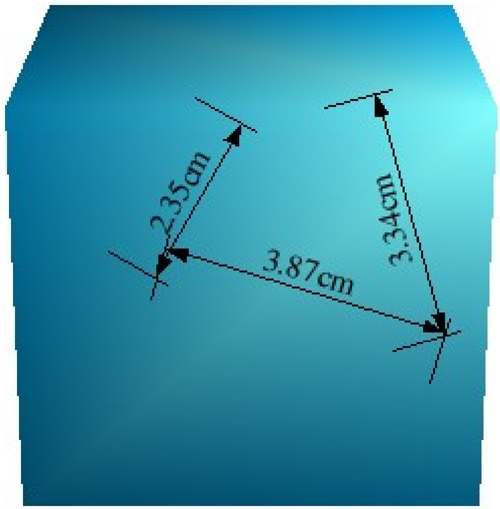
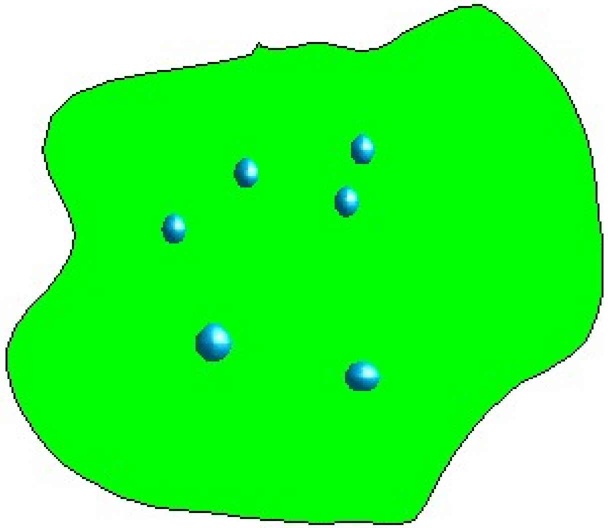
Corpi rigidi

Corpo rigido

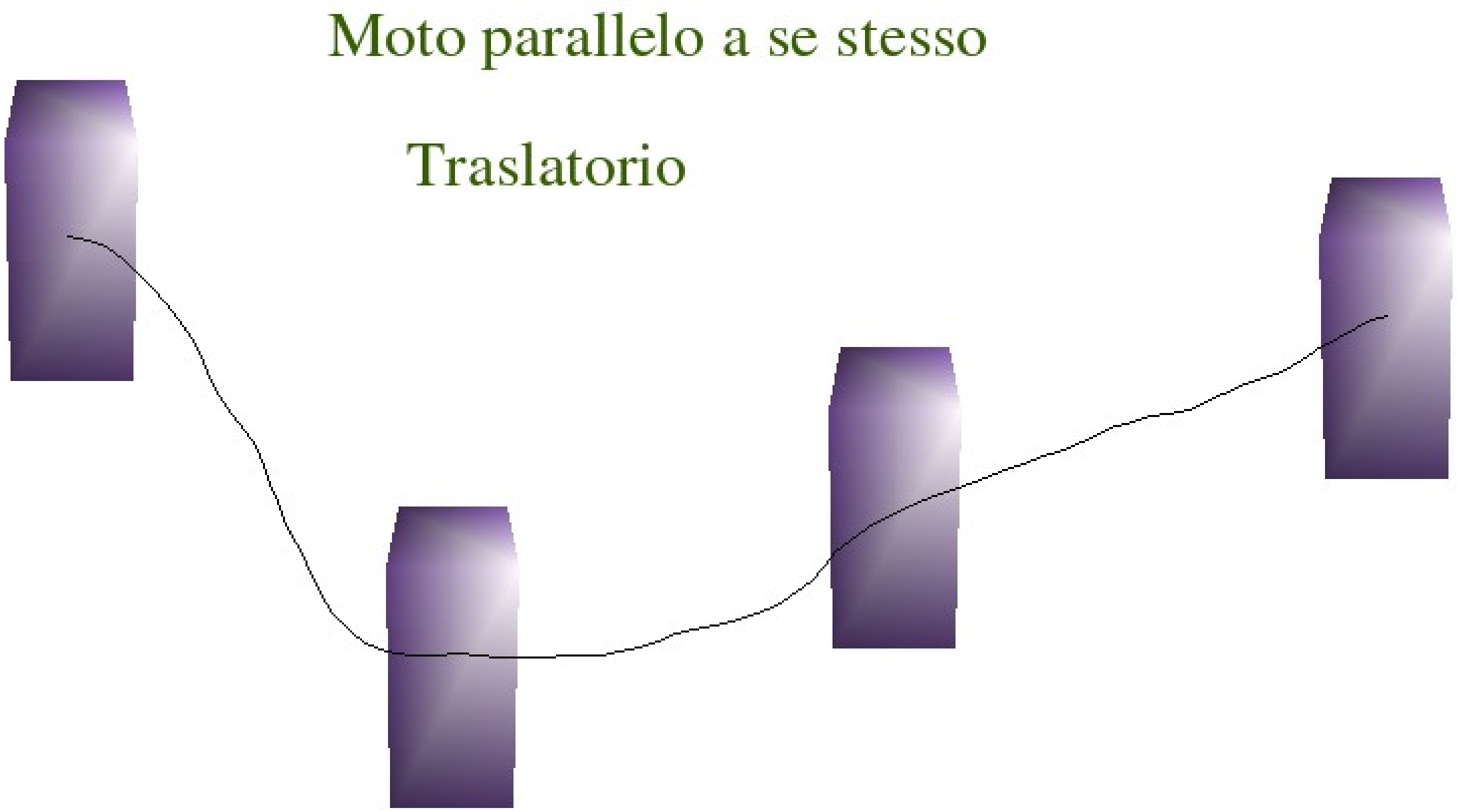


Sistema di punti



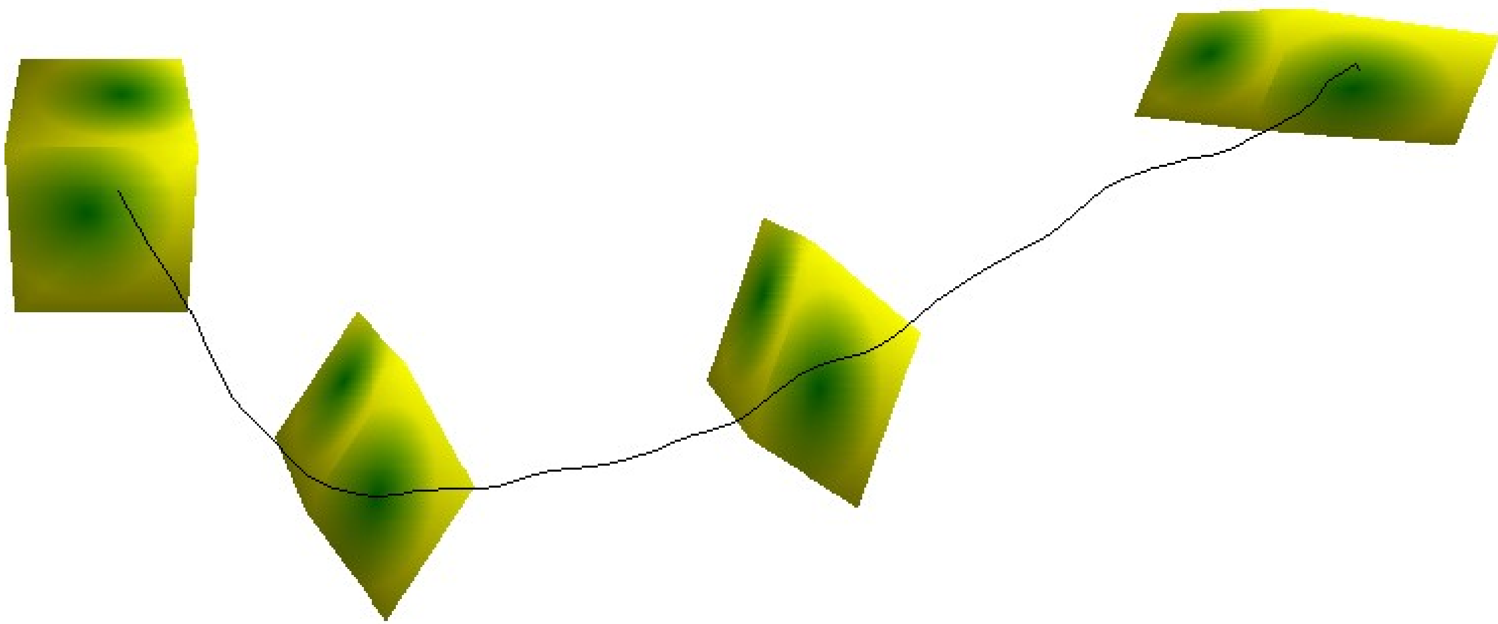
Le distanze tra i punti di un corpo rigido sono fisse

Corpi rigidi



Corpi rigidi

Moto roto-traslatorio

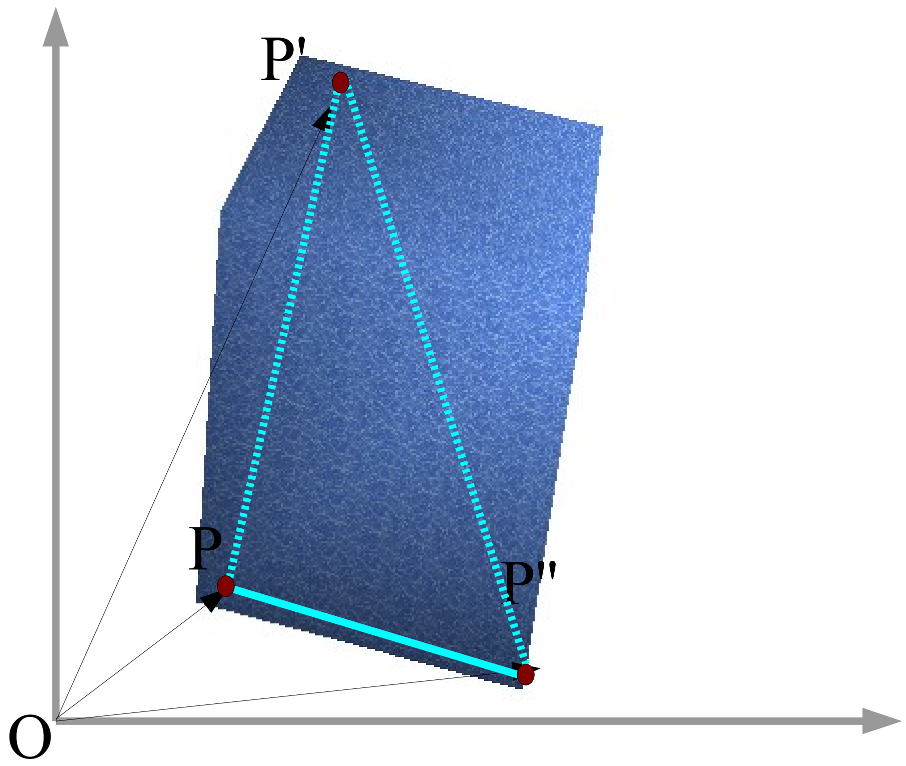


Corpi rigidi

gradi di liberta'

P ≡ (x,y,z)
P'≡ (x',y',z')
P"≡ (x",y",z")
9 parametri
3 relazioni di distanza
6 gradi di liberta'

Parametri liberi
3 coordinate
2 coseni direttori
1 angolo di orientazione

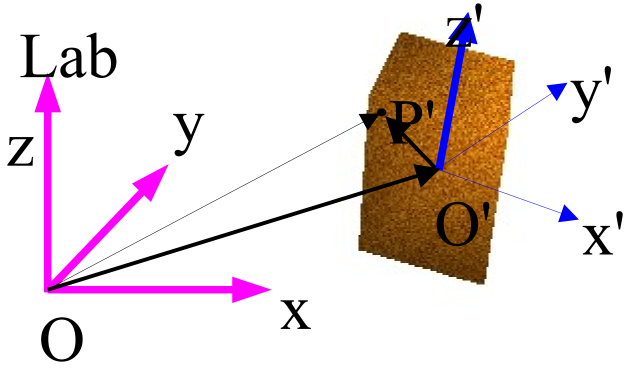


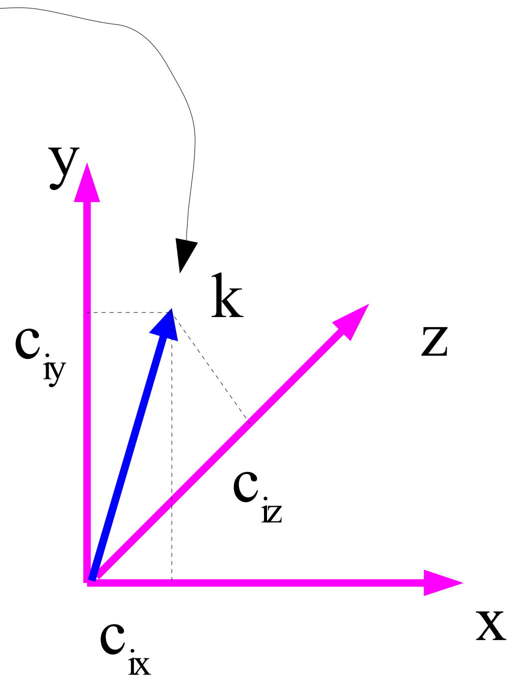
Corpi rigidi

Gradi di liberta'

P'O= O'O+PO'

P'O=O'O + ix'+jy'+kz'



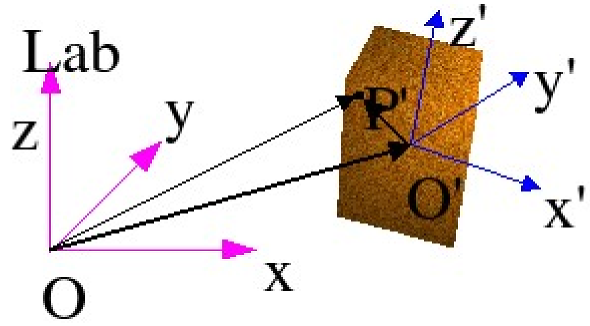


Corpi rigidi

Relazioni corpo <--> lab

P'O= O'O+PO'

P'O=O'O + ix'+jy'+kz'



Traslato

$$x = x_0' + x'$$

$$y = y_0' + y'$$

$$z = z_0' + z'$$

Rototraslato

$$\hat{i} \equiv (c_{ix}, c_{iy}, c_{iz})\, e\ simili$$

$$x = x_O' + c_{ix} x' + c_{jx} y' + c_{kx} z'$$

$$y = y_O' + c_{iy} x' + c_{jy} y' + c_{kz} z'$$

$$z = z_O' + c_{iz} x' + c_{jz} y' + c_{kz} z'$$

Corpi rigidi

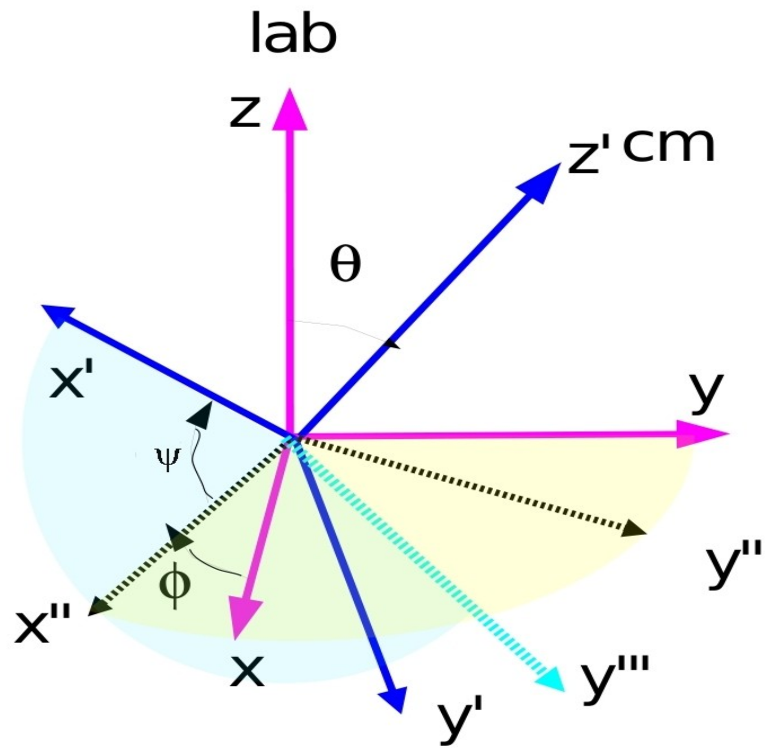
Relazioni corpo <--> lab

$$x = x_O' + c_{ix} x' + c_{jx} y' + c_{kx} z'$$

$$y = y_O' + c_{iy} x' + c_{jy} y' + c_{kz} z'$$

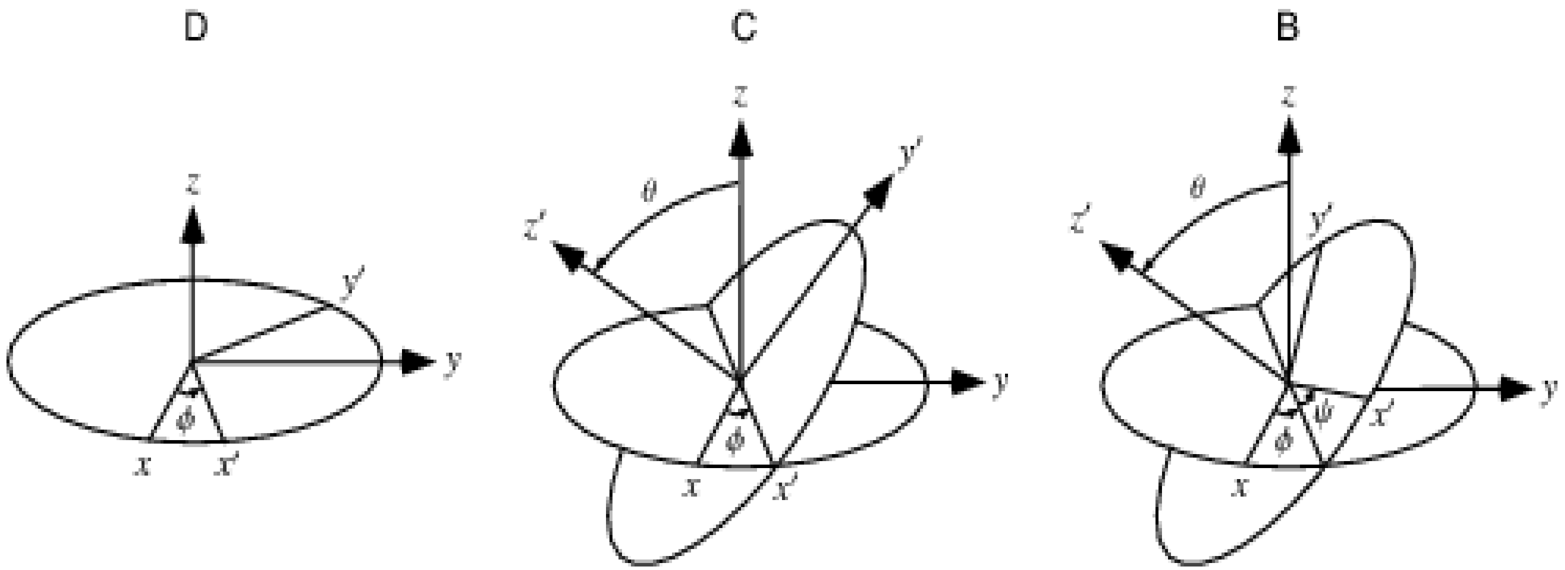
$$z = z_O' + c_{iz} x' + c_{jz} y' + c_{kz} z'$$

Operativamente
Ruoto di ϕ attorno a z
Ruoto di θ attorno a x''
Ruoto di ψ attorno a z'



$$\overrightarrow{P'O} = \overrightarrow{O'O} + R(\phi, \theta, \psi)\, \overrightarrow{P'O'}$$

Le tre rotazioni in ϕ,θ,ψ



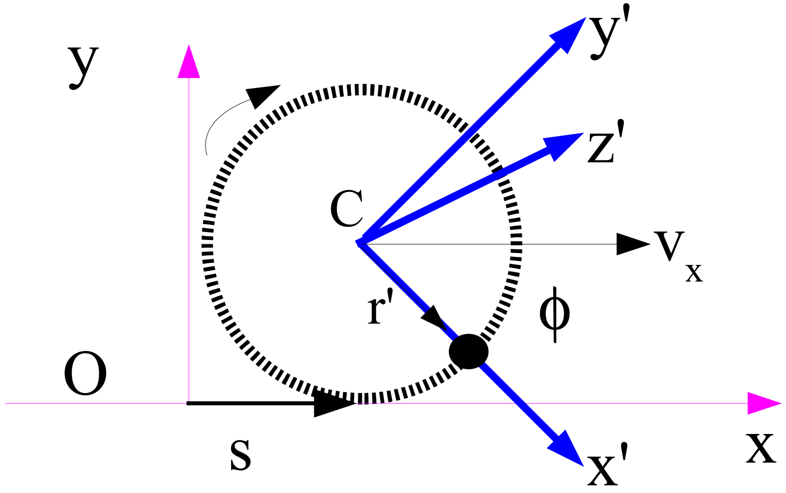
Gli angoli di Eulero

$$R(\phi,\theta,\psi)=\begin{vmatrix}\cos\psi & \sin\psi & 0\\ -\sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{vmatrix}\begin{vmatrix}1 & 0 & 0\\ 0 & \cos\theta & \sin\theta\\ 0 & -\sin\theta & \cos\theta\end{vmatrix}\begin{vmatrix}\cos\phi & \sin\phi & 0\\ -\sin\phi & \cos\phi & 0\\ 0 & 0 & 1\end{vmatrix}$$

R(ψ) * R(θ) * R(ϕ)

Corpi rigidi

Ruota in moto uniforme



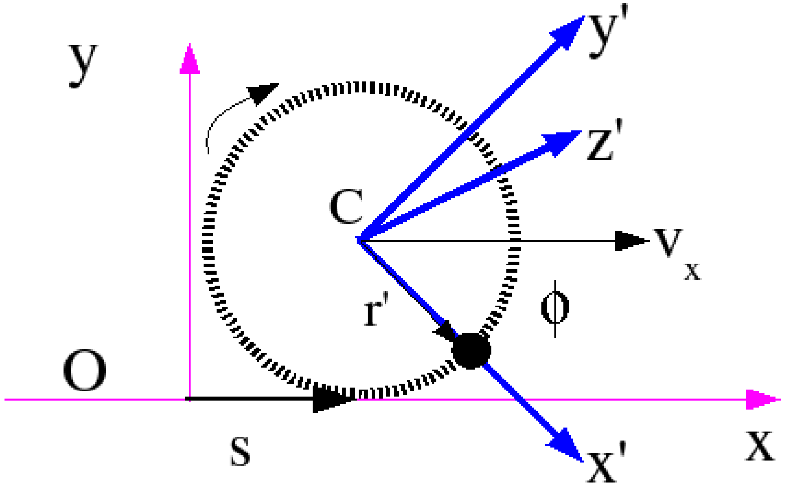
$$R(\phi)=\begin{vmatrix}\cos\phi & -\sin\phi & 0\\ \sin\phi & \cos\phi & 0\\ 0 & 0 & 1\end{vmatrix}$$

$$\vec{C}\equiv(0,r,0)$$

$$\vec{P}(t)=\vec{C}+\vec{v}_b t+R(\phi)\vec{r}\,'$$

Corpi rigidi

ciclode



$$= \vec{C} + \vec{v}_b t + R(\phi)\vec{r}\,'$$

$$\vec{C} \equiv (0,\ r,\ 0)$$

$$R(\phi) = \begin{vmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$x = v_c t + r\cos\phi$$
$$y = r + r\sin\phi$$
$$z = 0$$

v_x = costante

$\omega = -v_x t/r$

$$\dot{x} = v_c - r\dot{\phi}\sin\phi = v_c(1+\sin\phi)$$

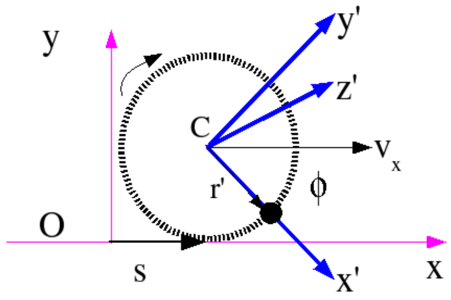
$$\ddot{x} = -r\dot{\phi}^2\cos\phi = -\frac{v_c^2}{r}\cos\phi$$

$$\dot{y} = r\dot{\phi}\cos\phi = -v_c\cos\phi$$

$$\ddot{y} = -r\dot{\phi}^2\sin\phi = -\frac{v_c^2}{r}\sin\phi$$

Corpi rigidi

moto rototraslatorio della ruota vettorialmente



Condizione iniziale

r vettore di P nel lab

r' vettore solidale con la ruota.

C=(0,r,0) Centro

$$\frac{d\vec{P}}{dt} = \dot{\vec{C}}_O + \vec{\omega} \wedge \vec{r} \qquad \omega \parallel z$$

$$\vec{r}\,' = R(\phi)\,\vec{r}\,' \equiv (r\cos\phi, r\sin\phi, 0)$$

$$\frac{d^2\vec{P}}{dt^2} = \vec{\omega} \wedge \vec{\omega} \wedge \vec{r} = -\omega^2\,\vec{r}$$

accelerazione verso il centro.