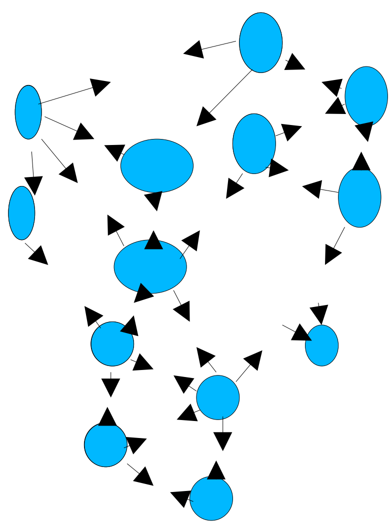
Sistemi estesi



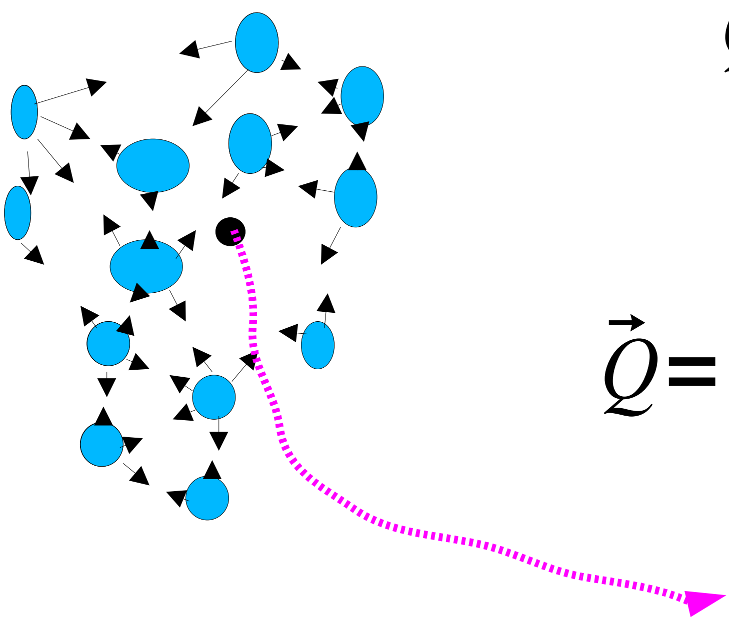
$$\vec{F}_i = m_i \vec{a}_i = m \frac{d\vec{v}_i}{dt} = \frac{d\vec{q}_i}{dt}$$

$$\sum_{i=1}^{N} \vec{F}_i = \sum_{i=1}^{N} m_i \vec{a}_i = \sum_{i=1}^{N} \frac{d\vec{q}_i}{dt} = \frac{d\vec{Q}}{dt}$$

dove $\vec{Q}$ *e' la quantita' di moto totale*

$$\vec{Q} = \sum_{i=1}^{N} \vec{q}_i = \sum_{i=1}^{N} m_i \vec{v}_i$$

Sistemi estesi
Centro di massa



$$\vec{Q}=\sum_{i=0}^{N}\vec{q}_i=\sum_{i=0}^{N}m_i\vec{v}_i$$ Quantita' di moto

e

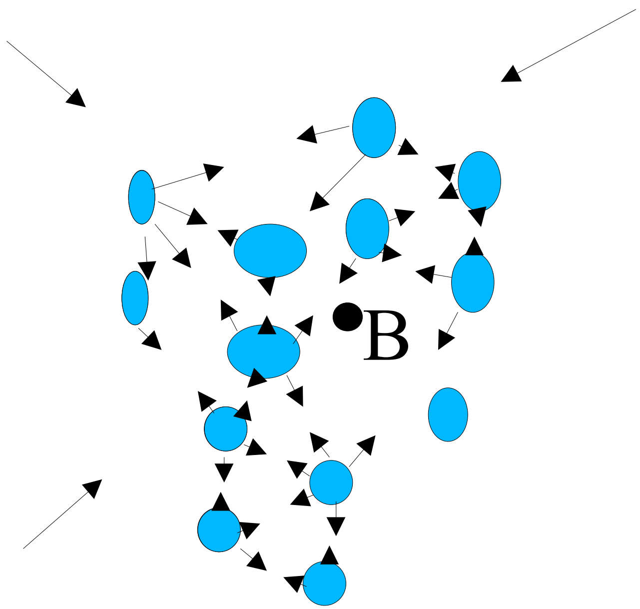
Centro delle masse

$$\vec{Q}=\sum_{i=0}^{N}m_i\frac{d\vec{r}_i}{dt}=\frac{d}{dt}\sum_{i=0}^{N}m_i\vec{r}_i$$

$$\vec{r}_B=\frac{1}{M}\sum_{i=0}^{N}m_i\vec{r}_i$$

$$\vec{Q}=\frac{d\,M\,\vec{r}_B}{dt}=M\,\vec{v}_B \quad quindisegue \rightarrow \quad F=\dot{\vec{Q}}$$

Sistemi estesi



$$\sum_{i=1}^{N}\left(\sum_{j=1}^{N}\vec{F}_{ji}+\vec{F}_{ei}\right)=\sum_{i=0}^{N} m_i a_i=\sum_{i=0}^{N}\frac{d\,q_i}{dt}=\frac{d\,\vec{Q}}{dt}$$

Forze interne si annullano identicamente e restano solo forze esterne

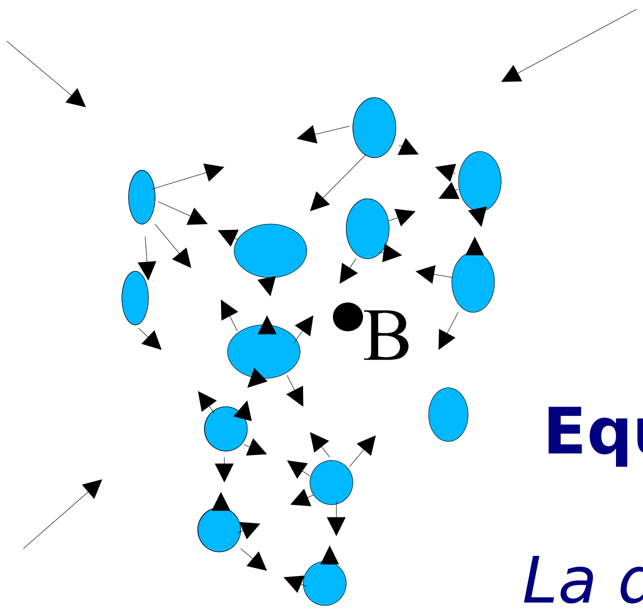
F_i esterne

F_e interne

poniamo $\vec{F}_e=\sum_{i=1}^{N_e}\vec{F}_{ei}$

si ottiene $\vec{F}_e=\frac{d\,\vec{Q}}{dt}$

Sistemi estesi



$$\vec{F}_e = \frac{d\vec{Q}}{dt}$$

Equazione cardinale dei sitemi

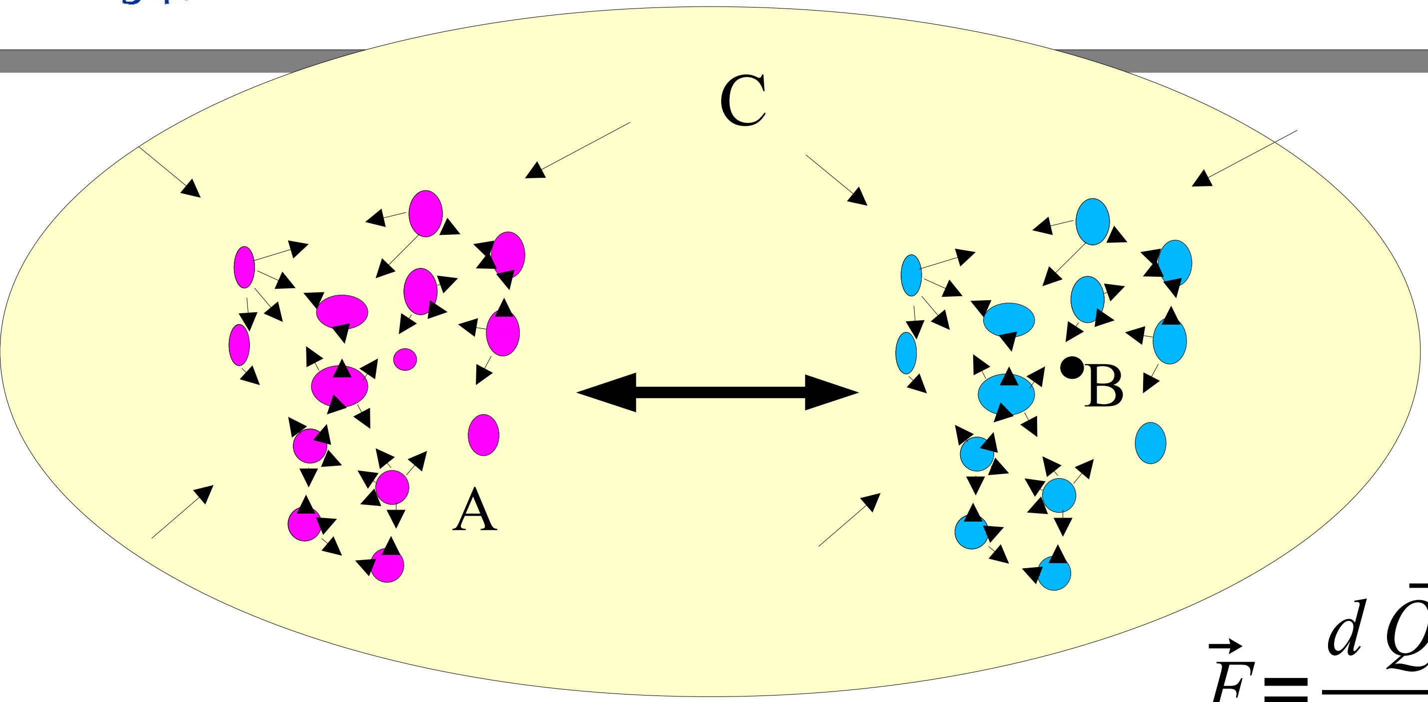
La derivata della quantità totale di un sistema è uguale alla risultante delle sole forze e

F_i esterne

F_e interne

Il centro di massa di un sistema di punti materiali si muove come un punto materiale di massa M soggetto alla forza $\mathbf{F}_e$.

Sistemi estesi



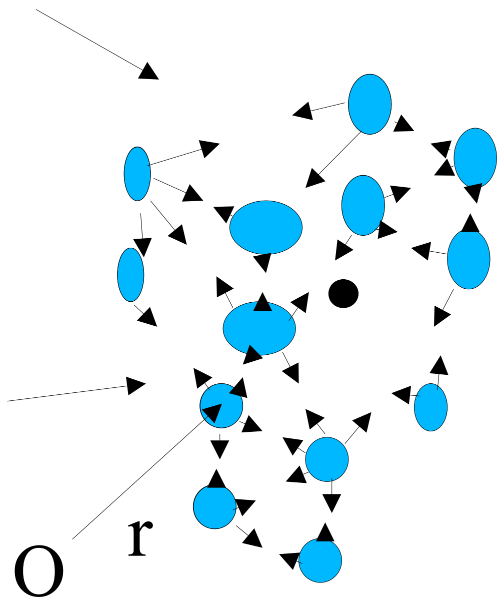
1. esterne per A e B : F_{eA} , F_{eB}
2. interne ad A : F_{iA}
3. interne a B : F_{iB}
4. di A su B : F_{AB}
5. di B su A : F_{BA}

$$\vec{F} = \frac{d\vec{Q}_A}{dt} + \frac{d\vec{Q}_B}{dt} = \frac{d\vec{Q}_C}{dt}$$

$$\vec{F} = \vec{F}_{eA} + \vec{F}_{eB} + \vec{F}_{iA} + \vec{F}_{iB} + \vec{F}_{AB} + \vec{F}_{BA}$$

$$\vec{F}_{eC} = \vec{F}_{eA} + \vec{F}_{eB} = \frac{d\vec{Q}_C}{dt}$$

Sistemi estesi
conservazione di Q



$$\vec{F}_e = \frac{d\vec{Q}}{dt}$$

Se $\qquad \vec{F}_e = 0$

$\longrightarrow$

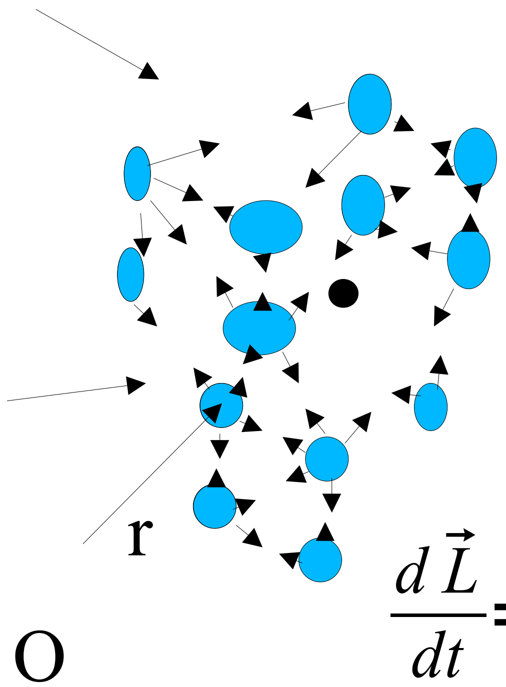
$$\vec{Q} = \vec{Cost.} \quad \rightarrow \quad \vec{v}_b = \vec{Cost}$$

Tre costanti

$$F_x = 0 \quad \rightarrow \quad Q_x = Cx \quad etcc....$$

Sistemi estesi

Momento angolare con polo fisso



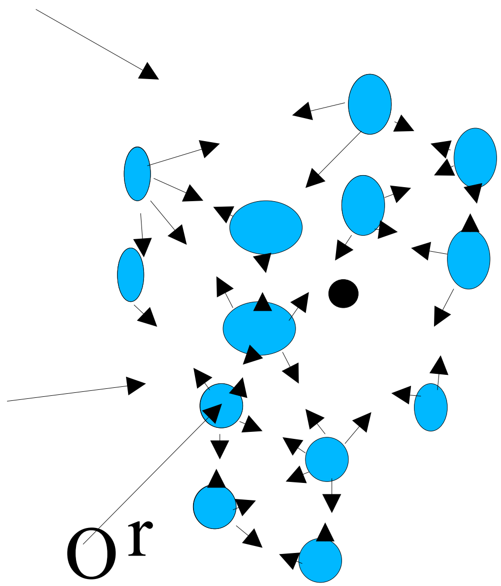
$$\vec{L}_i = \vec{r}_{io} \times m_i \vec{v}_i$$

$$\vec{L} = \sum_{i=1}^{N} \vec{L}_i = \sum_{i=1}^{N} \vec{r}_{io} \times m_i \vec{v}_i$$

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^{N} \frac{d}{dt}\left(\vec{r}_{io} \times m_i \vec{v}_i\right) = \sum_{i=1}^{N} \dot{\vec{r}}_{io} \times m_i \vec{v}_i + \vec{r}_{io} \times m_i \dot{\vec{v}}_i = \sum_{i=1}^{N} \vec{r}_{io} \times \vec{F}_i = \vec{M}$$

$$\longrightarrow \qquad \frac{d\vec{L}}{dt} = \vec{M}$$

Sistemi estesi
Polo fisso
Scomposizione delle forze

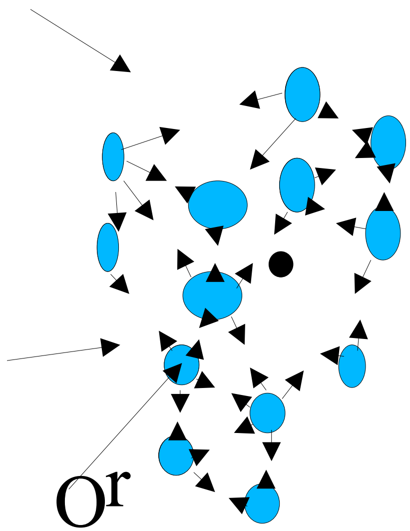


$$\frac{d\vec{L}}{dt}=\vec{M}=\sum_{i=1}^{N}\vec{r}_{io}\times\left(\sum_{k=1}^{N}\vec{F}_{ki}+\vec{F}_{ei}\right)=\cancel{\sum_{i=1}^{N}\sum_{k=1}^{N}\vec{r}_{io}\times\vec{F}_{ki}}+\sum_{i=1}^{N}\vec{r}_{io}\times\vec{F}_{ei}$$

$$\frac{d\vec{L}}{dt}=\sum_{i=1}^{N}\vec{r}_{io}\times\vec{F}_{ei}=\vec{M}_e$$

Contano solo le forze esterne!

Sistemi estesi
Polo mobile



$$\vec{L}_{\Omega}=\sum_{i=1}^{N}\vec{r}_{i\Omega}\times m_i\vec{v}_i$$

$$\frac{d\vec{L}_{\Omega}}{dt}=\sum_{i=1}^{N}\frac{d}{dt}\left(\vec{r}_{i\Omega}\times m_i\vec{v}_i\right)=\sum_{i=1}^{N}\dot{\vec{r}}_{i\Omega}\times m_i\vec{v}_i+\vec{r}_{i\Omega}\times m_i\dot{\vec{v}}_i$$

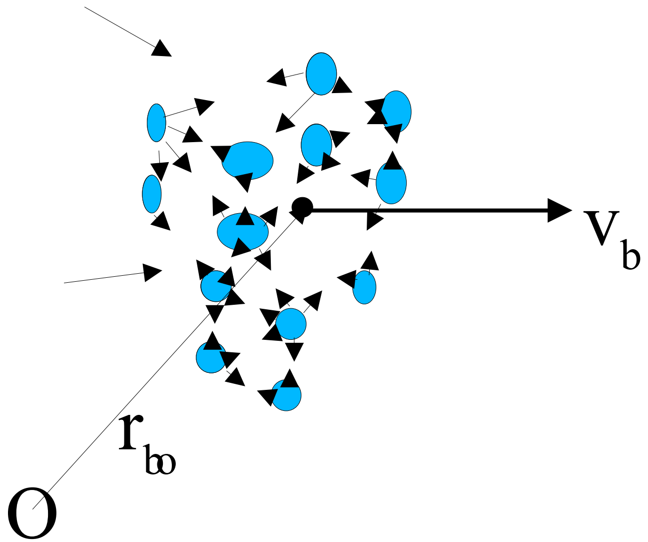
$$\vec{r}_{i\Omega}=\vec{r}_{iO}-\vec{r}_{\Omega O} \qquad \vec{v}_{i\Omega}=\vec{v}_i-\vec{v}_{\Omega O}$$

$$\longrightarrow \quad \frac{d\vec{L}_{\Omega}}{dt}=\vec{M}_e-\vec{v}_{\Omega}\times\vec{Q}$$

Sistemi estesi

In funzione delle cordinate e velocita' baricentrali

$$\vec{L}=\sum_{i=1}^{N}\vec{r}_{io}\times m_i\vec{v}_i=\sum_{i=1}^{N}(\vec{r}_{bo}+\vec{r}_i)\times m_i(\vec{v}_b+\vec{v}_{ib})=\vec{r}_{bo}\times\sum_{i=1}^{N}m_i\vec{v}_b+\sum_{i=1}^{N}(\vec{r}_{ib}\times m_i\vec{v}_{ib})$$



$$\vec{L}=\vec{L}_b+\vec{L}_r$$

$$\vec{L}_b=\vec{r}_{bo}\times M\,\vec{v}_b$$

$$\vec{L}_r=\sum_{i=1}^{N}((\vec{r}_{bi}+\vec{c})\times m_i\,\vec{v}_{bi})$$

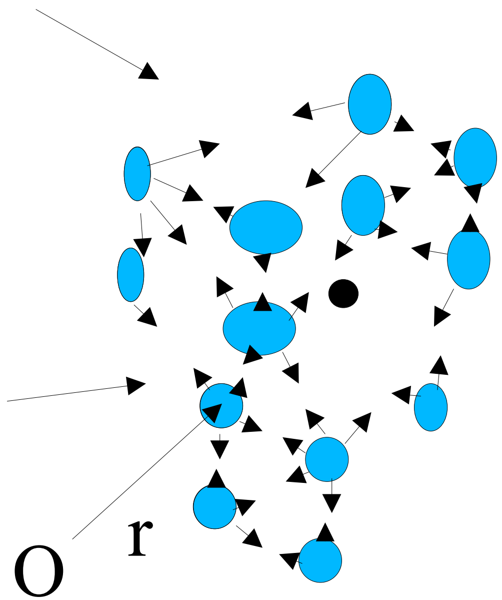
indipendente per traslazioni

$$\vec{M}_e=\sum_{i=1}^{N}\vec{r}_{io}\times\vec{F}_{ei}=\vec{r}_{bo}\times\sum_{i=1}^{N}\vec{F}_{ei}+\sum_{i=1}^{N}\vec{r}_{ib}\times\vec{F}_{ei}=\vec{r}_{bo}\times\vec{F}_e+\sum_{i=1}^{N}\vec{r}_{ib}\times\vec{F}_{ei}$$

Nota puo' essere nullo

Sistemi estesi

Conservazione momento angolare



$$\vec{M}_e = \frac{d\vec{L}}{dt}$$

Se $\vec{M}_e = 0$ $\longrightarrow$ $\vec{L} = \vec{Cost}$

$M_x = 0 \quad \rightarrow \quad L_x = Cx \quad etcc....$ Tre costanti