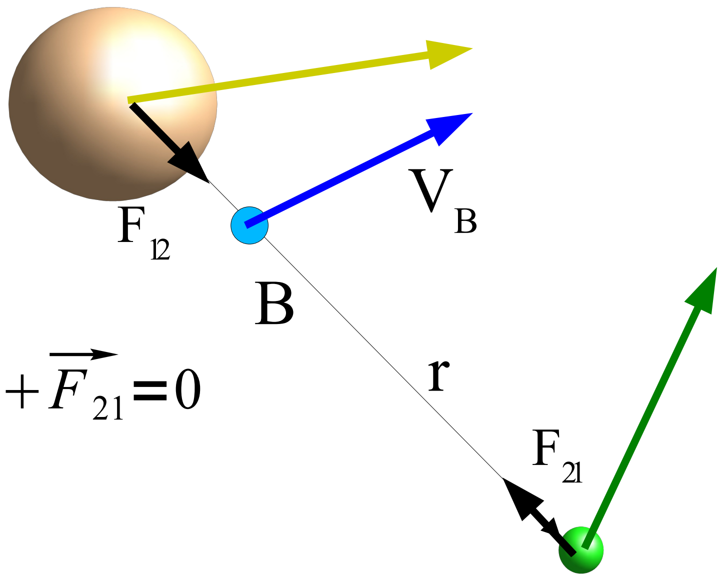
Moto centrale

due corpi

$$m_1\ddot{\vec{r}}_1=\overrightarrow{F}_{12}$$

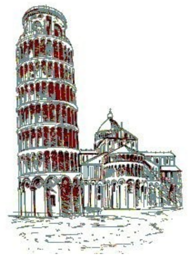
$$m_2\ddot{\vec{r}}_2=\overrightarrow{F}_{21}$$



$$m_1\ddot{\vec{r}}_1+m_2\ddot{\vec{r}}_2=\frac{d}{dt}(m_1\vec{v}_1+m_2\vec{v}_2)=\dot{\vec{Q}}=\overrightarrow{F}_{12}+\overrightarrow{F}_{21}=0$$

$$\vec{Q}=\frac{d}{dt}(m_1\vec{r}_1+m_2\vec{r}_2)=\frac{d}{dt}M\overrightarrow{R}_B=M\overrightarrow{V}_B$$

$$con\quad M=(m_1+m_2)\qquad \overrightarrow{R}_B=\frac{(m_1\vec{r}_1+m_2\vec{r}_2)}{(m_1+m_2)}\quad e\quad \overrightarrow{V}_B=\frac{\vec{Q}}{M}$$



Moto centrale

due corpi

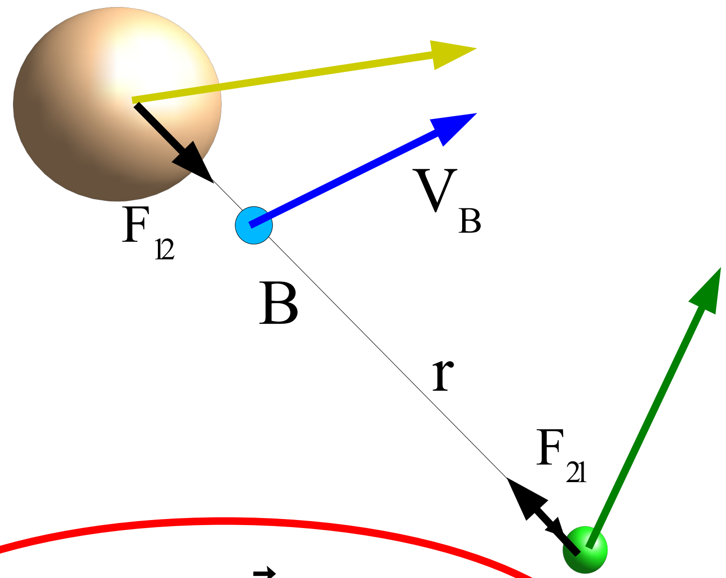
$$m_1\ddot{\vec{r}}_1=\vec{F}_{12}$$

$$m_2\ddot{\vec{r}}_2=\vec{F}_{21}$$

$$m_1 m_2(\ddot{\vec{r}}_1-\ddot{\vec{r}}_2)=m_2\vec{F}_{12}-m_1\vec{F}_{21}=M\vec{F}_{12}$$

$$\vec{r}=\vec{r}_1-\vec{r}_2 \qquad \mu=\frac{m_1 m_2}{m_1+m_2}$$

$$\mu\ddot{\vec{r}}=\vec{F}_{21}$$

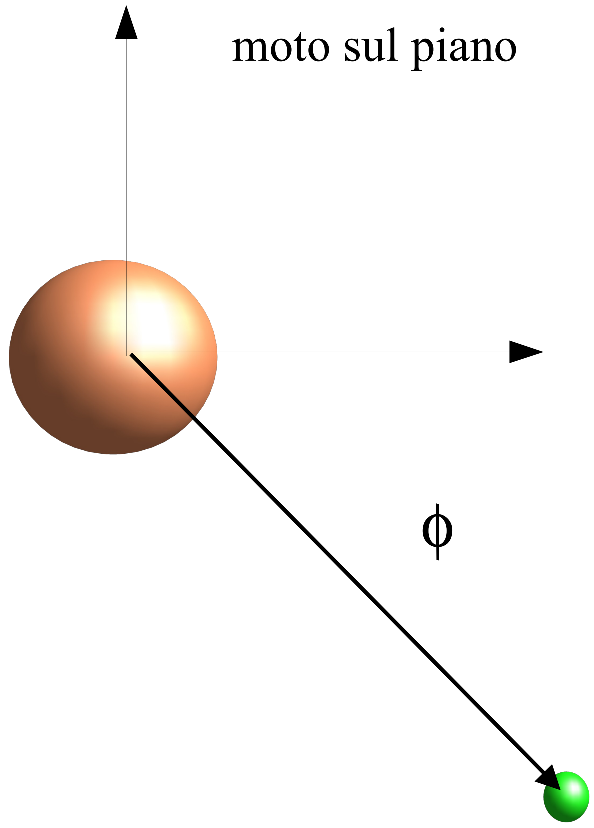


$$\mu\ddot{\vec{r}}=F(r)\frac{\vec{r}}{r}=F(r)\hat{r}$$

Moto centrale

gravita'

$$F(r)\hat{r} = -\frac{GMm}{r^2}\hat{r} \qquad U(r) = -\frac{GMm}{r}$$



$$E = \frac{1}{2} m v^2 - \frac{GMm}{r} \qquad \vec{L} = \vec{r} \times m\vec{v}$$

$$E = \frac{1}{2} m(\dot{r}^2 + r^2\dot{\phi}^2) - \frac{GMm}{r} \qquad L = m r^2 \dot{\phi}$$

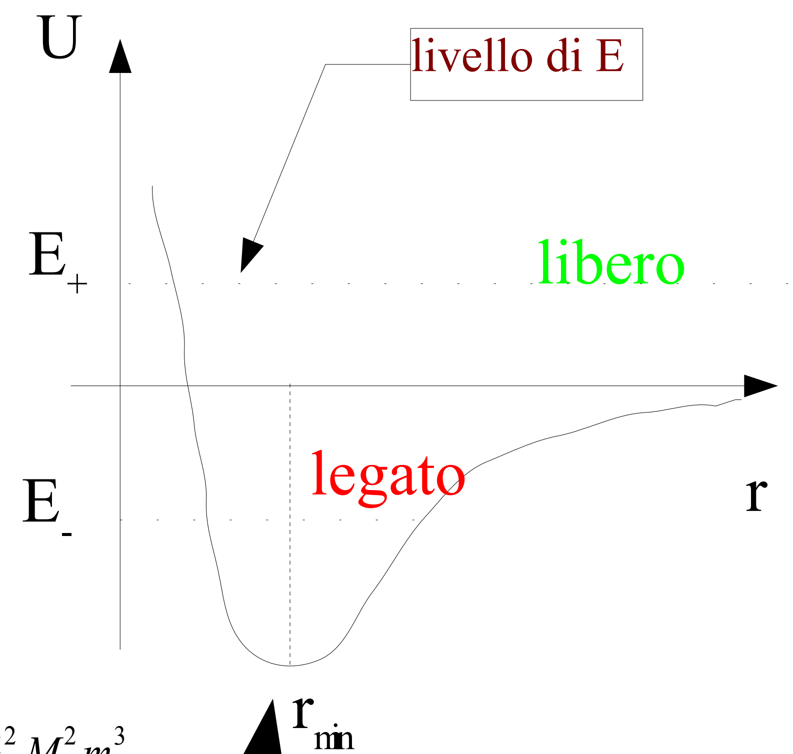
Moto centrale
potenziale efficace

$$\dot{\phi} = \frac{L}{m r^2}$$

$$E = \frac{1}{2} m \dot{r}^2 + \frac{L^2}{2\mathrm{m}\, r^2} - \frac{GMm}{r}$$

$$E = \frac{1}{2} m \dot{r}^2 + U(r)_{eff.}$$

con $\quad U(r)_{eff.} = \frac{L^2}{2\mathrm{m}\, r^2} - \frac{GMm}{r}$



$L = mr^2 \dot{\phi} \quad$ *con* $\quad r_{min} = \frac{L^2}{GMm^2} \quad$ *segue* $\quad \dot{\phi} = \frac{G^2 M^2 m^3}{L^3}$

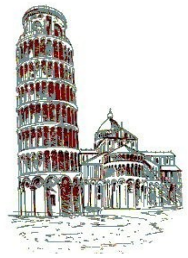
Moto centrale
equazione del moto

$$\frac{dE}{dt} = \frac{d\left(\frac{1}{2} m \dot{r}^2 + \frac{L^2}{2\mathrm{m}\, r^2} - \frac{GMm}{r}\right)}{dt} = 0$$

da cui

$$m \ddot{r} = \frac{L^2}{m r^3} - \frac{GMm}{r^2}$$

che e' un po' difficile ??



Moto centrale

vettore di Lenz

$$\vec{A} = \vec{v} \times \vec{L} - GMm\hat{r}$$

$$\vec{A}\cdot\vec{r} = A\,r\cos\phi = \vec{r}\cdot\vec{v}\times\vec{L} - GMmr = \frac{L^2}{m} - GMmr$$

$$r = \frac{\frac{L^2}{m}}{GMm + A\cos\phi} \equiv \frac{p}{1+e\cos\phi}$$

Equazione della traiettoria

$$p = \frac{L^2}{GMm^2} \qquad e \equiv \frac{A}{GMm}$$

Moto centrale

equazione traiettoria

$$r=\frac{p}{1+e\cos\phi} \qquad r+er\cos\phi=p \qquad r=p-ex$$

$$x^2+y^2=(p-ex)^2 \quad segue \quad (1-e^2)x^2+y^2+2pex=p^2$$

$$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1$$

$$a=\frac{p}{1-e^2} \qquad b=\frac{p}{\sqrt{1-e^2}} \qquad x_0=\frac{ep}{1-e^2} \qquad y_0=0$$

$$a=\frac{p}{1-e^2}\equiv\frac{GMm}{2|E|}$$

$$b=\frac{p}{\sqrt{1-e^2}}=a\sqrt{1-e^2}\equiv\sqrt{\frac{L^2}{2m|E|}}$$

$$\frac{dA}{dt}=\frac{L}{2m}=costante$$

$$T=2\pi ab\frac{m}{L}$$

$$T=2\pi ab\frac{m}{L}=2\frac{\pi}{\sqrt{GM}}a^{\frac{3}{2}}$$

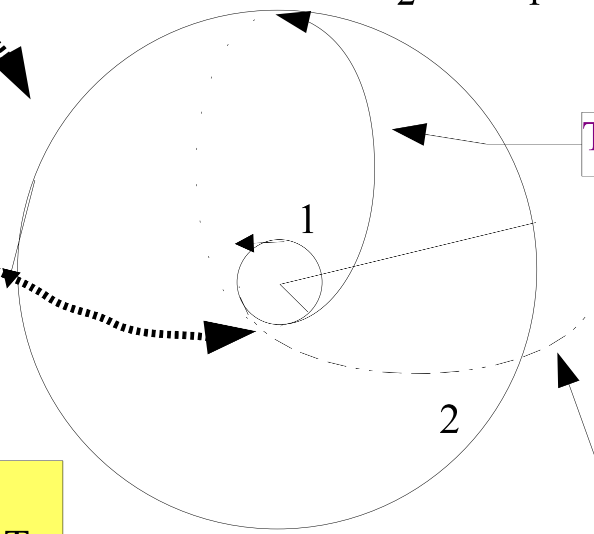
III di Keplero

Moto centrale
Orbite

Energia nell'orbita circolare

$$E = -\frac{1}{2}\frac{GMm}{r_c}$$

$$E_2 - E_1 = -\frac{1}{2}\frac{GMm}{r_2} + \frac{1}{2}\frac{GMm}{r_1}$$



T orbita di transizione

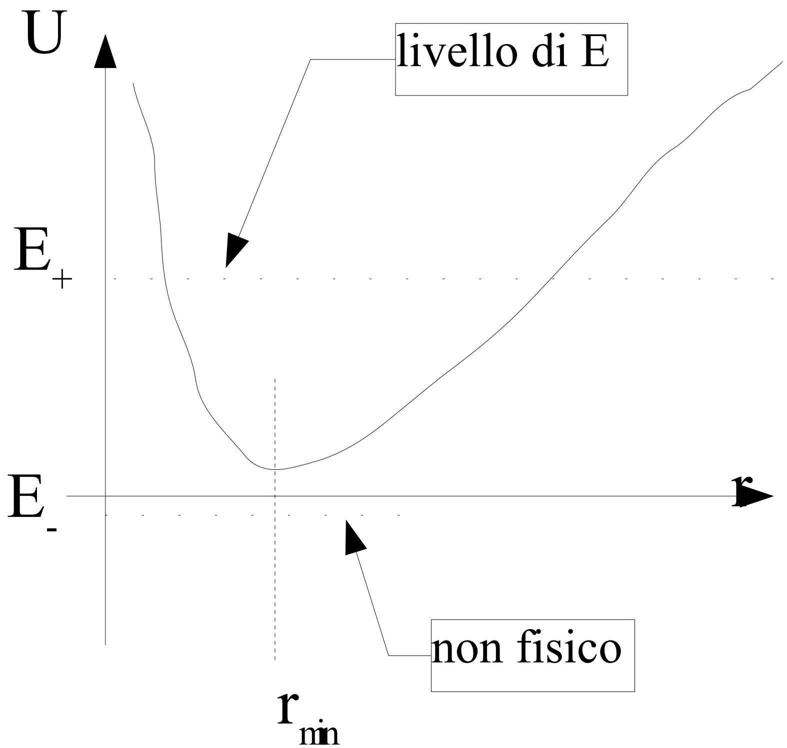
orbita non economica

???????
- Trovare l'energia dell'orbita T
- L dell'orbita T
- Centro dell'orbita T

Orbita geostazionaria.. cosa e' ???

Moto centrale oscillatore

$$F(r)\hat{r} = -k\vec{r} \qquad U(r) = \frac{1}{2}k r^2$$



$$E = \frac{1}{2}m\dot{r}^2 + U(r)_{eff.}$$

con $$U(r)_{eff.} = \frac{L^2}{2\mathrm{m}\, r^2} + kr^2$$