Campo di forze centrale

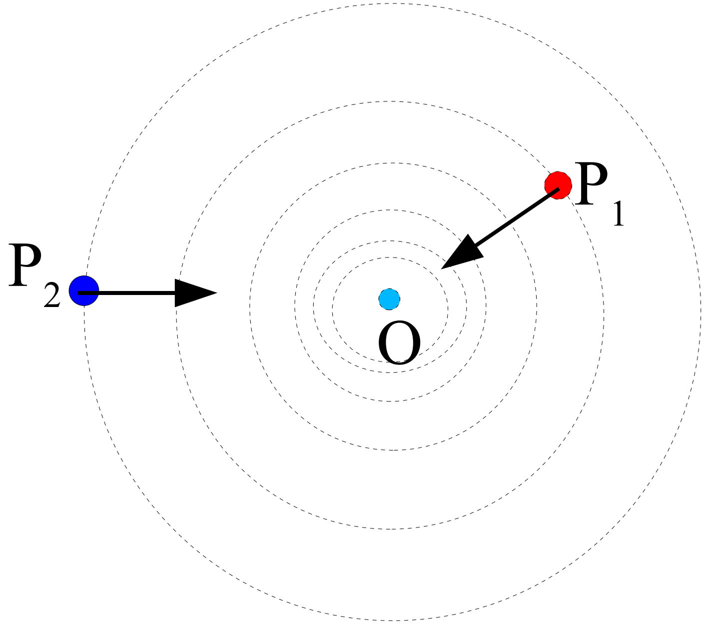
$$\vec{F}(\vec{R}) = f(r)\,\hat{R}$$

$$\vec{R} = \vec{P} - \vec{O}$$

$$\vec{F} \propto -\frac{M_T\, m}{R^2}\,\hat{R}$$

$$\vec{F} = -k\,r\,\hat{R}$$

$$\vec{f} \propto \frac{Q\,q}{R^2}\,\hat{R}$$

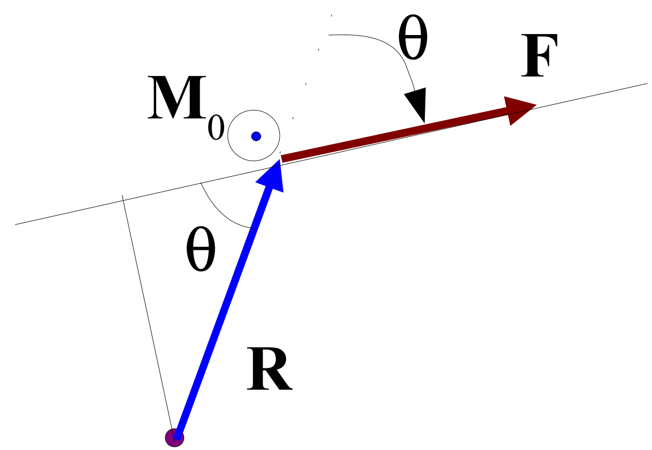


Momenti

Momento della Forza <==> Momento della quantita' di moto

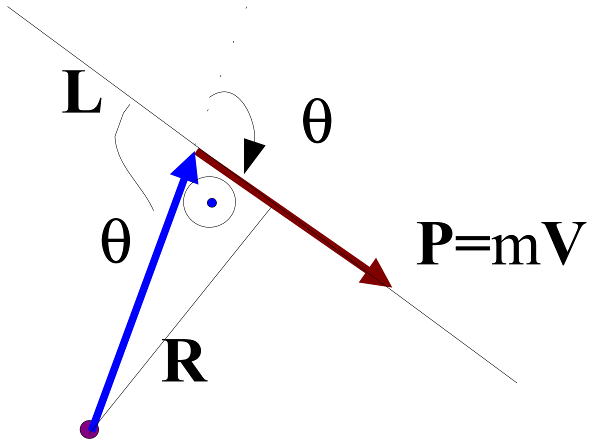
Della forza

$$\vec{M} = \vec{R} \wedge \vec{F} = r\, f \sin\theta\, \hat{n}$$



Della q. di moto

$$\vec{L} = \vec{R} \wedge \vec{P} = \vec{R} \wedge m\vec{v} = m\, r\, v \sin\theta\, \hat{n}$$



Momenti

Momento della Forza <==> Momento della quantita' di moto

$$\vec{M}=\vec{R}\wedge\vec{F}\quad\equiv r f \sin\theta\,\hat{n}$$

$$\vec{L}=\vec{R}\wedge\vec{P}=\vec{R}\wedge m\vec{v}\quad\equiv m r v \sin\theta\,\hat{n}$$

c'e' un legame?

$$\frac{d\vec{L}}{dt}=\frac{d(\vec{R}\wedge m\vec{v})}{dt}=\dot{\vec{R}}\wedge m\vec{v}+\vec{R}\wedge m\ddot{\vec{R}}$$

e' nulla !
poiche' dr/dt = v ed ‘O’ e' fisso

$$ricordando\ \vec{F}=m\vec{a}\qquad \frac{d\vec{L}}{dt}=\vec{R}\wedge\vec{F}=\vec{M}$$

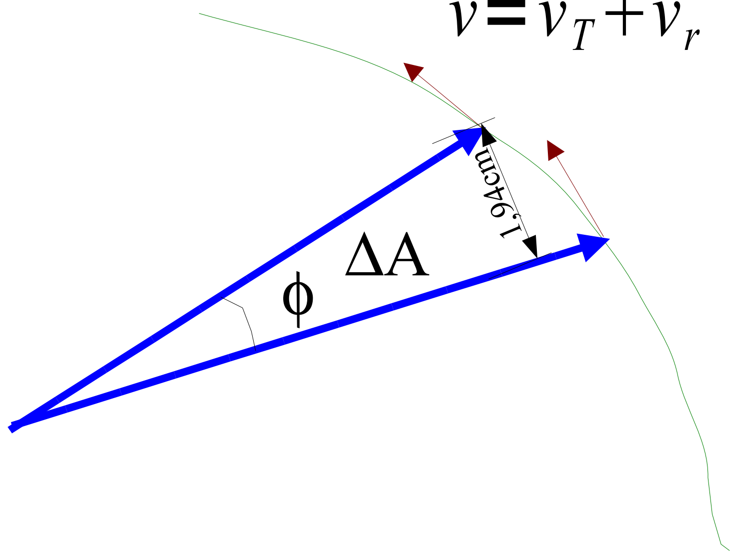
$$\dot{\vec{L}}=\vec{M}\qquad equazione\ canonica$$

Campo centrale

Velocita' areolare

$$Vel.areolare = lim_{\Delta T \to 0} \frac{\Delta A}{\Delta t} = lim_{\Delta T \to 0} \frac{r^2}{2} \frac{\Delta \phi}{\Delta t} = \frac{r^2}{2} \dot{\phi} = \frac{r}{2} v \sin\theta = \frac{L}{2\mathrm{m}}$$

$$\vec{v} = \vec{v}_T + \vec{v}_r \quad v_T = r\dot{\phi}$$



Campo centrale

Oscillatore tridimensionale

E' una forza centrale

$$\vec{F} = -K\vec{r}$$

$$\vec{M} = -\vec{r} \wedge k\vec{r} = 0$$

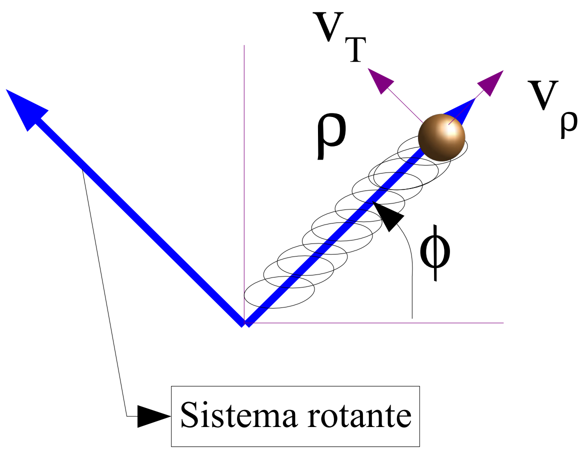
$$\dot{\vec{L}} = 0 \quad quindi \; \vec{L} = \vec{L}_0 \; costante$$

Campo centrale

Oscillatore tridimensionale

Moto piano ==> coordinate polari

$$\vec{v} = \vec{v}_\rho + \vec{v}_T$$



$$m\ddot{\rho} = -k\rho + m\rho\dot{\phi}^2$$

$$m\rho\ddot{\phi} = -2\mathrm{m}\dot{\rho}\dot{\phi}$$

$$\vec{L} = \vec{r} \wedge m\vec{v} = \vec{\rho} \wedge m\vec{v}_T = m\rho\dot{\phi}\vec{\rho} \wedge \hat{T}$$

$$L = m\rho^2\dot{\phi} = costante$$

$$m\ddot{\rho} = -k\rho + \frac{L^2}{m\rho^3}$$

Campo centrale
Costanti del moto

$$\vec{M}=0 \quad \rightarrow \quad \vec{L}=Costant$$
$$\vec{F}=0 \quad \rightarrow \quad \vec{p}=Costant$$

$$F_x=0 \quad \rightarrow \quad p_x=costx.$$
$$F_y=0 \quad \rightarrow \quad p_y=costy.$$
$$F_z=0 \quad \rightarrow \quad p_z=costz.$$

$$E=T+V \quad ???$$