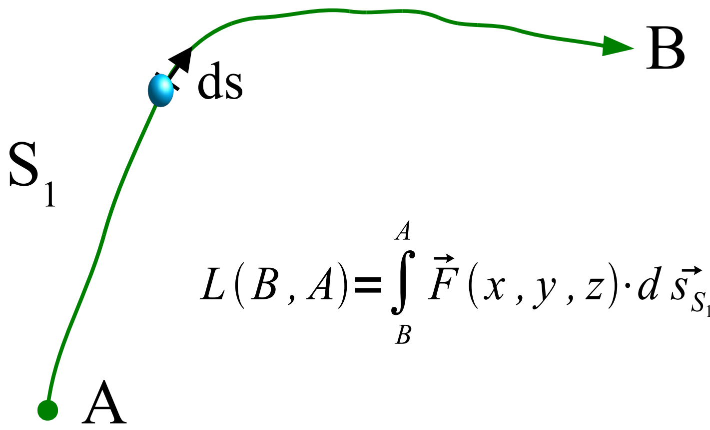
Energia

forze conservative

$$\lim \sum_{A}^{B} \vec{F}(x,y,z)\cdot \Delta \vec{s}_{S_1} = \int_{A}^{B} \vec{F}(x,y,z)\cdot d\vec{s}_{S_1} = L(A,B)$$

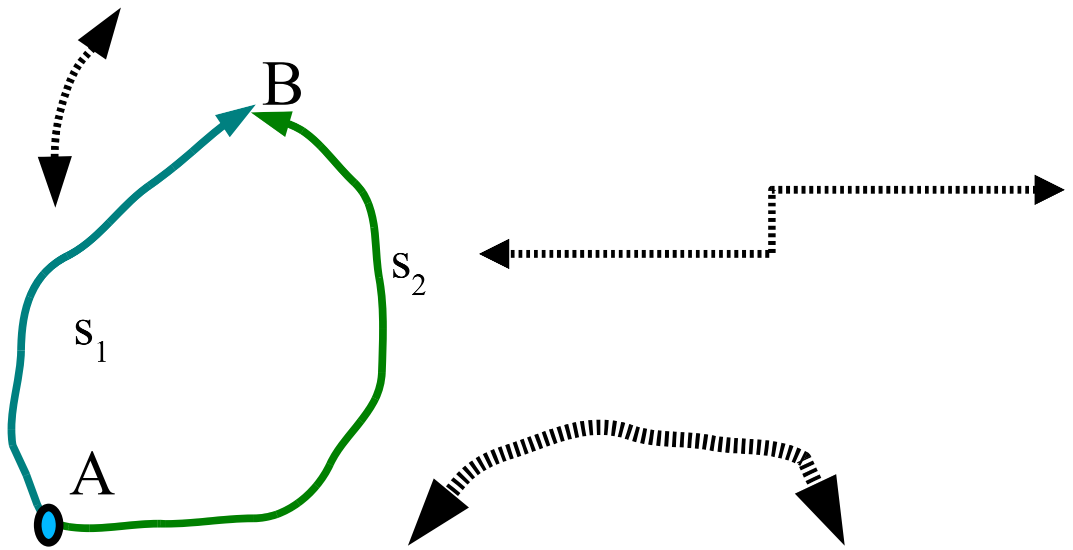
F dipende solo dal punto !

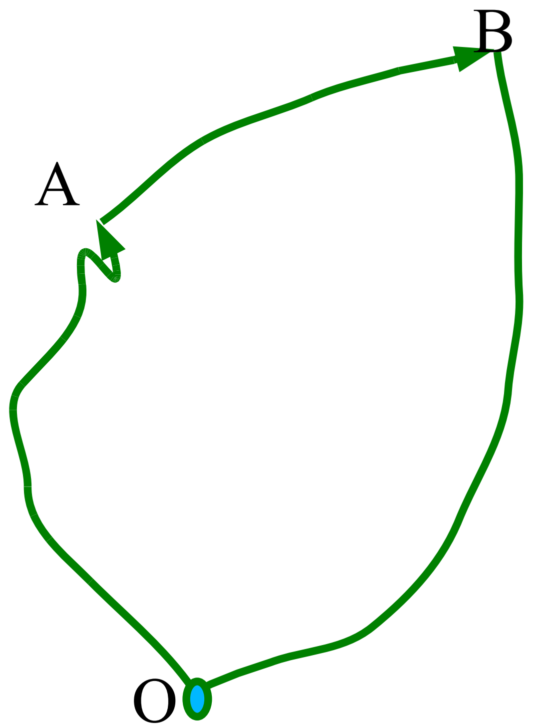


$$L(B,A) = \int_{B}^{A} \vec{F}(x,y,z)\cdot d\vec{s}_{S_1} = -\int_{A}^{B} \vec{F}(x,y,z)\cdot d\vec{s}_{S_1} = -L(A,B)$$

Energia
forze conservative

Supponi:
$L_{s1}(A,B) = L_{s2}(A,B)$





$$\oint_{S_1+S_2} \vec{F}(x,y,z)\cdot d\vec{s} = 0$$

$$\int_A^B \vec{F}(x,y,z)\cdot d\vec{s} = L(A,B) = L(O,B) - L(O,A)$$

Energia
forze conservative e potenziale

$$\int_A^B \vec{F}(x,y,z)\cdot d\vec{s} = L(A,B) = L(O,B) - L(O,A)$$

$$\int_{P_1}^{P_2} F(x,y,z)\cdot d\vec{s} = \Phi(P_2) - \Phi(P_1)$$

$$potenziale \qquad U(P) = -\Phi(P)$$

$$\int_A^B \vec{F}\cdot d\vec{s} = L(A,B) = U(A) - U(B) = \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2$$

$$E = U(A) + \frac{1}{2} m v_A^2 = U(B) + \frac{1}{2} m v_B^2$$

Costante del moto

Energia

forze conservative e potenziale

$L_{s1}(A,B) = L_{s2}(A,B)$

$$\oint_S \vec{F}(x,y,z)\cdot d\vec{s} = 0$$

$$\int_{P_1}^{P_2} F(x,y,z)\cdot d\vec{s} = \Phi(P_2) - \Phi(P_1)$$

$$U(P) = -\Phi(P)$$

Energia
potenziale e forza

$$\vec{F}\cdot\Delta\vec{s}=F_x\Delta s_x+F_y\Delta s_y+F_z\Delta s_z=L(\vec{P},\vec{P}+\Delta\vec{s})=U(\vec{P})-U(\vec{P}+\Delta\vec{s})$$

$$F_z\Delta z=U(x_P,y_P,z_P)-U(x_P,y_P,z_P+\Delta z)$$

$$F_z=lim_{\Delta z\to 0}\frac{U(x_P,y_P,z_P)-U(x_P,y_P,z_P+\Delta z)}{\Delta z}=-\frac{\partial U}{\partial z}$$

Energia
potenziale e forza

$$\vec{F}\cdot d\vec{s} = -\left(\frac{\partial U}{\partial x}ds_x + \frac{\partial U}{\partial y}ds_y + \frac{\partial U}{\partial z}ds_z\right) = -\vec{\nabla}U\cdot d\vec{s}$$

con

$$\vec{\nabla}U \equiv \left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)U \quad = \quad \left(\frac{\partial U}{\partial x},\frac{\partial U}{\partial y},\frac{\partial U}{\partial z}\right)$$

$$\vec{F} = -\vec{\nabla}U$$

Energia

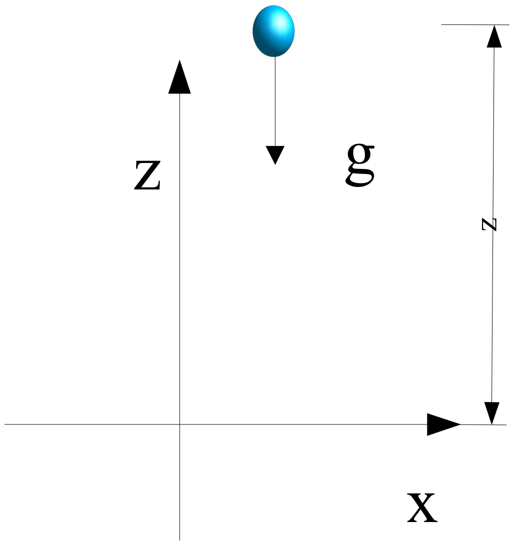
forze conservative

$$\int_0^z m\vec{g}\cdot d\vec{s} = -\int_0^z m\, g\, dz = 0 - m\, g\, z = L(0,z) = U(0) - U(z)$$

da cui

$$U(z) = m\, g\, z \qquad \textit{il poteziale}$$

$$\frac{1}{2} m\, v^2 + m\, g\, z = E_0$$



$$\frac{1}{2} m\, v^2 = E_0 = m\, g\, h \qquad \textit{da cui} \qquad v = \sqrt{2 g h}$$