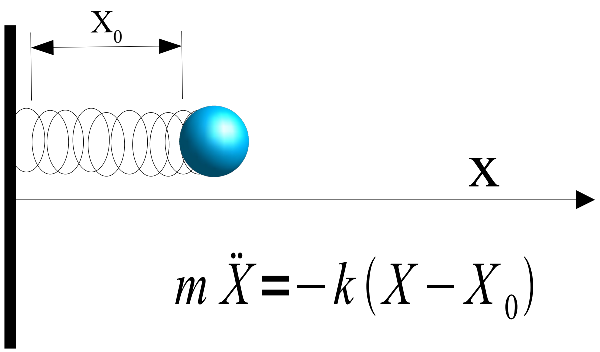
Oscillatore armonico

unidimensionale



$$m\ddot{X} = -k(X - X_0)$$

$$X(0) = 0 \quad \dot{x}(0) = v_0$$

$$X = X_0 + \frac{v_0}{\omega}\sin(\omega t)$$

$$x(0) = A_0 \quad \dot{x}(0) = 0$$

$$X = X_0 + A_0\cos(\omega t)$$

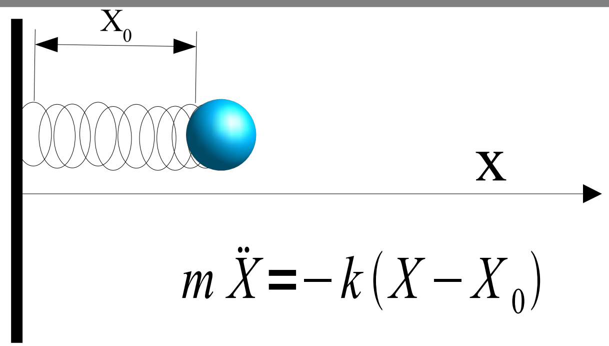
$$x = X - X_0$$

$$m\ddot{x} = -kx \quad segue \quad \ddot{x} = -\omega^2 x \quad con \quad \omega = \sqrt{\frac{k}{m}}$$

$$x = A\sin(\omega t + \phi) \qquad \dot{x} = A\omega\cos(\omega t + \phi)$$

Oscillatore

unidimensionale



$$m\ddot{X} = -k(X - X_0)$$

$$A\sin(\phi) = A_0 \quad \text{per la posizione iniziale}$$

$$A\omega\cos(\phi) = v_0 \quad \text{per la velocità iniziale}$$

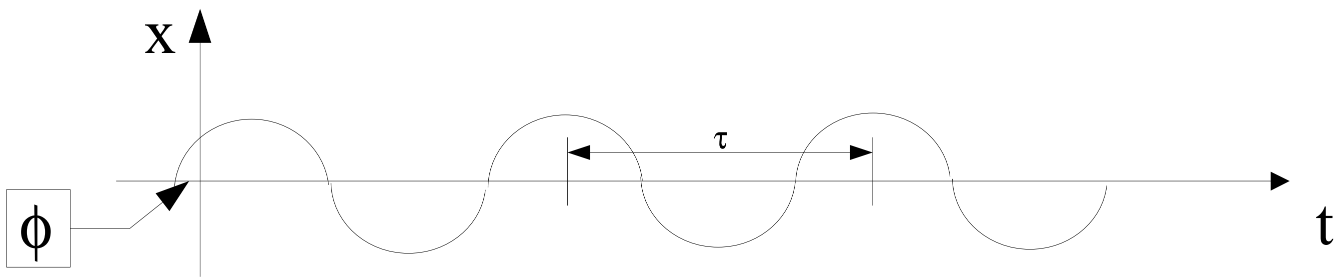
$$A = \frac{1}{\omega^2}\sqrt{\omega^2 A_0^2 + v_0^2} \qquad e \quad \tan\phi = \frac{\omega A_0}{v_0}$$

$$\sin(\phi) = \frac{\omega A_0}{\sqrt{v_0^2 + \omega^2 A_0^2}} \qquad \cos(\phi) = \frac{v_0}{\sqrt{v_0^2 + \omega^2 A_0^2}}$$

$$X = X_0 + A\sin(\omega t + \phi) = X_0 + A\sin(\omega t)\cos(\phi) + A\cos(\omega t)\sin(\phi)$$

segue

$$X = X_0 + \frac{v_0}{\omega}\sin(\omega t) + A_0\cos(\omega t)$$



Oscillatore

Costante del moto

$$\int_A^B -kx\,dx = \frac{1}{2}kx_A^2 - \frac{1}{2}kx_B^2 = \frac{1}{2}m\,v_B^2 - \frac{1}{2}m\,v_A^2$$

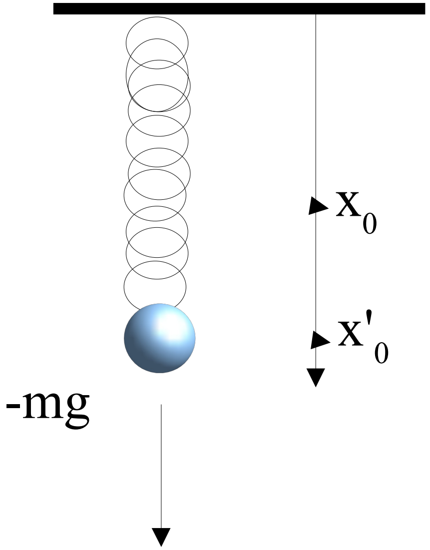
ovvero la somma della energia cinetica piu' l'energia di posizione (potenziale)

$$E = \frac{1}{2}m\,v_A^2 + \frac{1}{2}kx_A^2 = \frac{1}{2}m\,v_B^2 + \frac{1}{2}kx_B^2$$

e' costante

$$\frac{1}{2}k\,A_0^2 = \frac{1}{2}k\,x^2 + \frac{1}{2}m\,v^2$$

Oscillatore armonico
in un campo di forza costante



$$m\ddot{X} = -k(X - X_0) + mg$$

$$m\ddot{X} = -kX + kX_0 + mg = -k(X - X') \qquad X'_0 = X_0 + \frac{mg}{k}$$

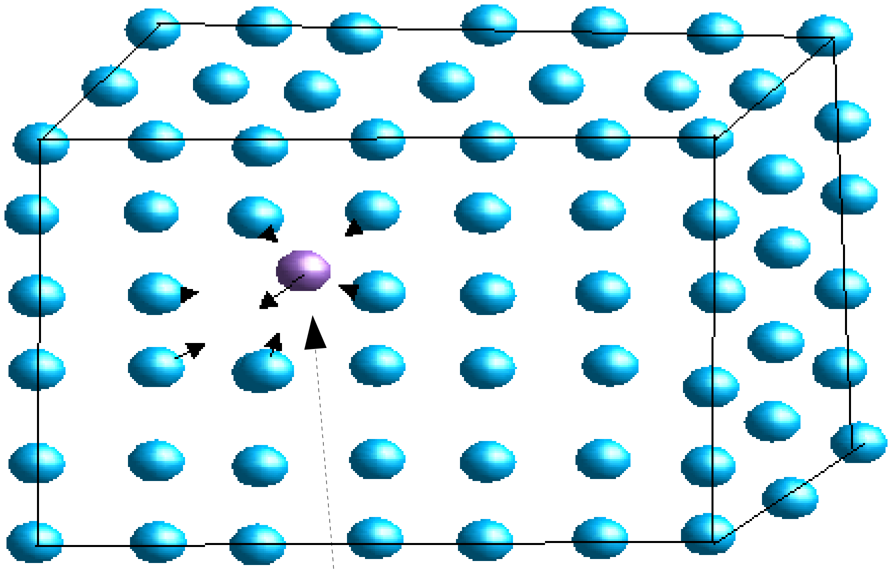
$$m\ddot{X} = -k(X - X'_0)$$

$$X = X'_0 + \frac{v_0}{\omega}\sin(\omega t) + A_0\cos(\omega t)$$

Oscillatore armonico

perche' e' importante

Struttura cristallina



Forza di richiamo

$$F_x(x,y,z)=F_x(0)+\frac{\delta F_x}{\delta x}x+......=-kx$$

$$F_y(x,y,z)=F_y(0)+\frac{\delta F_y}{\delta y}y+......=-ky$$

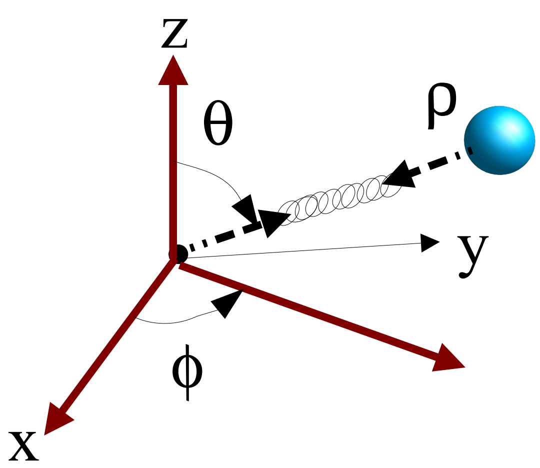
$$F_z(x,y,z)=F_z(0)+\frac{\delta F_z}{\delta z}z+......=-kz$$

o vettorialmente

$$\vec{F}=-k\vec{r}$$

Oscillatore armonico

tridimensionale



$$m\ddot{\vec{r}} = -k\vec{r}$$

che equivale alle tre equazioni indipenden[ti]

$$m\ddot{x} = -kx$$

$$m\ddot{y} = -ky$$

$$m\ddot{z} = -kz$$

con soluzioni armoniche note sono

$$x = A_x \sin(\omega t + \phi_x)$$

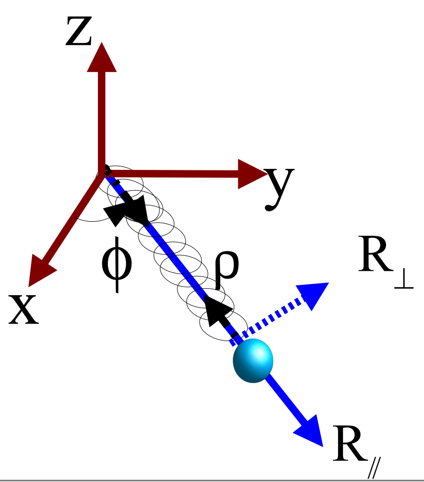
$$y = A_y \sin(\omega t + \phi_y)$$

$$z = A_z \sin(\omega t + \phi_z)$$

==> moti indipendenti, ma stessa frequenza => moti chiusi
==> se A_z e' nullo ==> moto piano composizione dei moti x e y

Oscillatore armonico

in coordinate polari sul piano



$$m\ddot{\vec{R}}=\vec{F}(\rho,\phi)$$

$$\vec{R}\equiv(\rho\cos\phi,\rho\, sin\phi)\equiv\rho(\cos\phi, sin\,\phi)\equiv\rho\hat{R}$$

$con\,\omega=\dot{\phi}\,\hat{z}\ e\ \dot{\omega}=\ddot{\phi}\,\hat{z}$ *la velocita' e'*

$$\dot{\vec{R}}=\dot{\rho}\,\hat{R}+\vec{\omega}\wedge\vec{R}$$

l'accelerazione, dopo un po' di conti...

$$\ddot{\vec{R}}=\ddot{\rho}\,\hat{R}+\dot{\rho}\,\vec{\omega}\wedge\hat{R}+\dot{\vec{\omega}}\wedge\vec{R}+\vec{\omega}\wedge(\dot{\rho}\,\hat{R}+\vec{\omega}\wedge\vec{R})$$

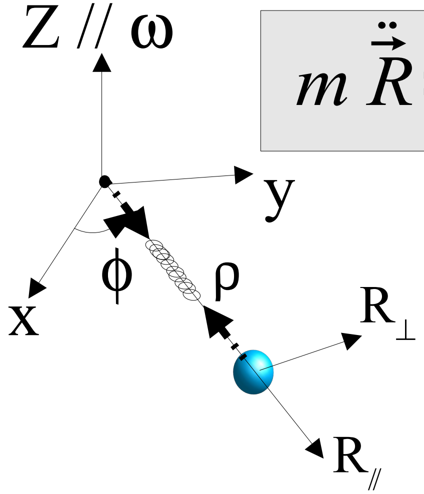
$$\ddot{\vec{R}}=(\ddot{\rho}-\omega^2\rho)\,\hat{R}+(\dot{\omega}\,\rho+2\,\omega\,\dot{\rho})\,\hat{\omega}\wedge\hat{R}$$

$$\ddot{\vec{R}}=(\ddot{\rho}-\omega^2\rho)\,\hat{R}+(\dot{\omega}\,\rho+2\,\omega\,\dot{\rho})\,\hat{T}\qquad \hat{T}\perp\hat{R}$$

$$m\,\ddot{\rho}-m\,\rho\,\dot{\phi}^2=F_{F\,\|\vec{R}}(r,\phi)\qquad \frac{1}{\rho}\frac{d\,m\,\rho^2\dot{\phi}}{dt}=F_{F\,\perp\,\vec{R}}(\rho,\phi)$$

Oscillatore armonico

Equazione nel sistema ruotante



$$m\,\ddot{\vec{R}} = -k\,\vec{R}$$

Coriolis

$$\vec{a}_c = 2\,\vec{\omega}\wedge\vec{v}$$

da cui la forza apparen

$$\vec{F}_c = -2\,m\,\vec{\omega}\wedge\vec{v}$$

Forza elastica $= -k\,\rho$

Forza apparente centrifuga $= m\,\rho\,\dot{\phi}^2$

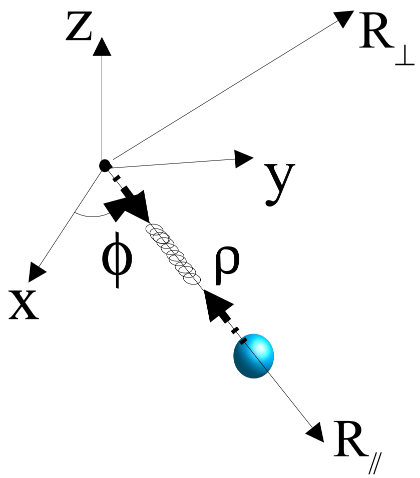
Forza apparente di Coriolis $= -2\mathrm{m}\,\dot{\rho}\,\dot{\phi}$

da cui

$$m\,\ddot{\rho} = -k\,\rho + m\,\rho\,\dot{\phi}^2$$

$$m\,\rho\,\ddot{\phi} = -2\mathrm{m}\,\dot{\rho}\,\dot{\phi}$$

Oscillatore armonico



$$\boxed{m\ddot{\vec{R}} = -k\vec{R}}$$

$$m\ddot{\rho} - m\rho\dot{\phi}^2 = -k\rho$$

$$\frac{1}{\rho}\frac{d\,m\rho^2\dot{\phi}}{dt} = 0 \quad \textit{ovvero}$$

$$m\rho^2\dot{\phi} = L \quad \textit{costante!}$$

$$m\ddot{\rho} = -k\rho + \frac{L^2}{m\rho^3}$$

1) L=0 $\qquad m\ddot{\rho} = -k\rho$

2) L >0 e r e' costante $\quad -k\rho + m\rho\dot{\phi}^2 = 0 \qquad \dot{\phi} = \omega = \sqrt{\frac{k}{m}}$