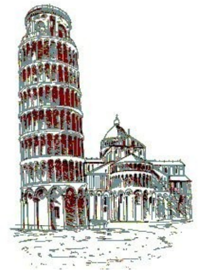


Cinematica
caduta libera

accelerazione costante

$$\vec{a}_0 \equiv (0,0,a_0)$$

$$x(0)=y(0)=z(0)=0 \quad \vec{v}=0$$

$$v_z=a_0 t \quad z(t)=\frac{1}{2}a_0 t^2$$

$$\vec{g} \equiv (0,0,-g)$$

$$x(0)=y(0)=0 \quad z(0)=h=50m\,t \quad \vec{v}_{z0}=0$$

$$v_z=-g\,t \quad z(t)=-\frac{1}{2}g\,t^2+h$$

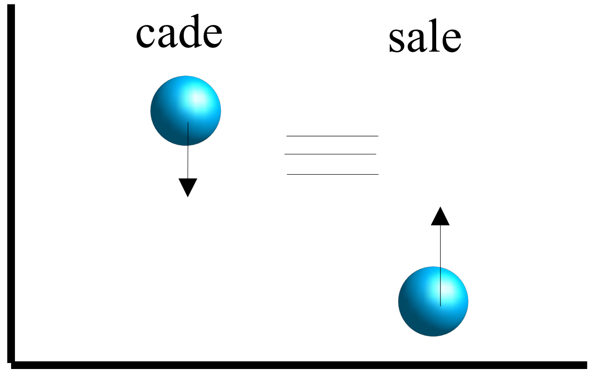
$$t=\sqrt{\frac{2h}{g}}=3.19s \quad v_z=-\sqrt{2hg}=31.3m/s$$

$$x(0)=y(0)=0 \quad z(0)=0 \quad v_{z0}=31.3m/s$$

$$v_z=-g\,t+v_{z0}$$

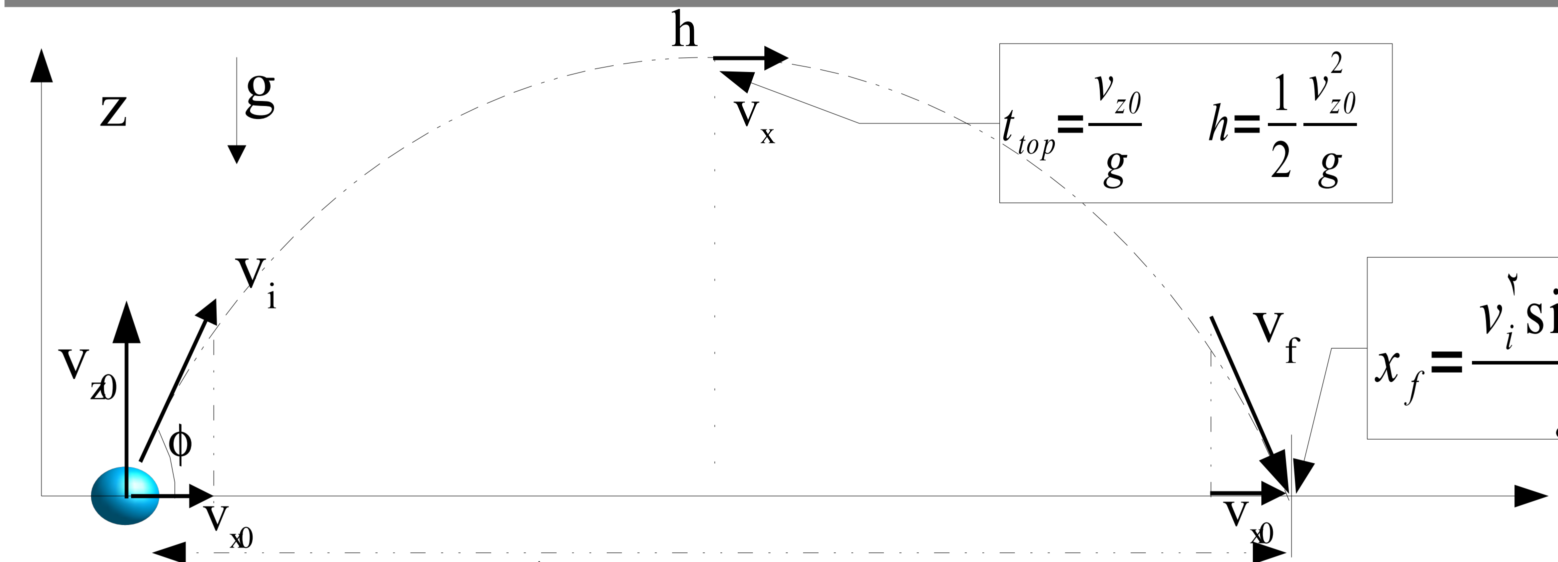
$$z(t)=-\frac{1}{2}g\,t^2+v_{z0}t$$

$$t_{top}=\frac{v_{z0}}{g}=3.19s \quad h=\frac{1}{2}\frac{v_{z0}^2}{g}=50m$$



Cinematica

traiettoria parabolica



$$t_{top} = \frac{v_{z0}}{g} \qquad h = \frac{1}{2}\frac{v_{z0}^2}{g}$$

$$x_f = \frac{v_i^2 \sin 2\phi}{g}$$

$$x = v_{x0} t$$

$$y = 0$$

$$z = -\frac{1}{2} g t^2 + v_{z0} t$$

$$z = -\frac{1}{2}\frac{g}{v_{x0}^2} x^2 + \frac{v_{z0}}{v_{x0}} x$$

Cinematica

moto circolare promemoria

$$\vec{R}=(\rho\cos\phi\,,\rho\sin\phi)=\rho\,\hat{R}$$

$$\hat{R}=(\cos\phi\,,\sin\phi)$$

$$\frac{d\,\hat{R}}{d\,\phi}=(-\sin\phi\,,\cos\phi)=\hat{T}$$

$$\frac{d\,\hat{R}}{dt}=\frac{d\,\hat{R}}{d\,\phi}\frac{d\,\phi}{dt}=\dot{\phi}\,\hat{T}=\omega\,\hat{T}$$

dove $\omega=\dot{\phi}=\dfrac{d\,\phi}{dt}$

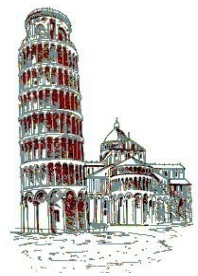
gia' noto

Corollario

$$ds=\rho\,d\,\phi$$

$$\frac{d\,\vec{R}}{ds}=\frac{\rho\,d\,\hat{R}}{\rho\,d\,\phi}=(-\sin\phi\,,\cos\phi)=\hat{T}$$

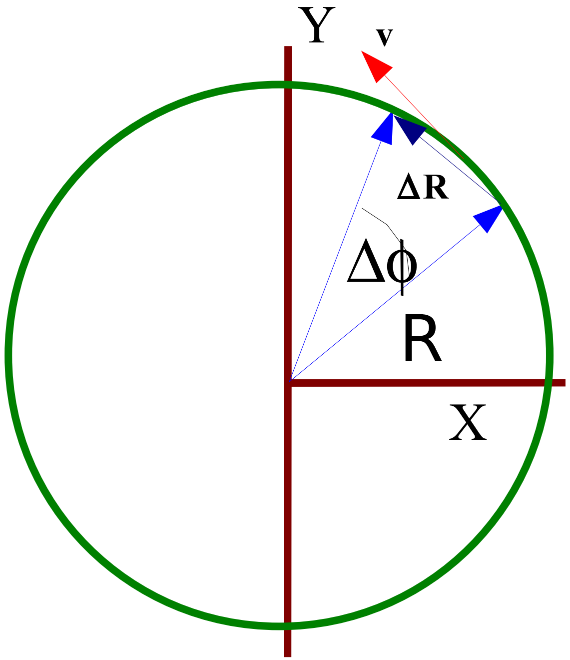
$$\frac{d\,\vec{R}}{dt}=\frac{\rho\,d\,\hat{R}}{d\,\phi}\frac{d\,\phi}{dt}=\rho\,\dot{\phi}\,\hat{T}=\rho\,\omega\,\hat{T}$$



Cinematica

moto circolare non uniforme

Circolare



$$\vec{R}(t) = \rho \hat{R} \qquad con \qquad \hat{R} = (\cos(\phi(t)), \sin(\phi(t)))$$

$$la\ velocita'\ \vec{v} = \lim \frac{\Delta \vec{R}}{\Delta t} \quad for\ \Delta t ==> 0 \qquad segue ==>$$

$$\vec{v}(t) = \frac{d\vec{R}}{dt} = \rho \frac{d\hat{R}}{dt} = \rho \frac{d\hat{R}}{d\phi}\frac{d\phi}{dt} = \rho \dot{\phi} \widehat{R_n} = \rho \omega \widehat{R_n} = \rho \omega \hat{T}$$

$$dove \qquad \frac{d\hat{R}}{d\phi} = \widehat{R_n} = \hat{T}$$

... formalmente non differisce dal caso di velocita' uniforme.

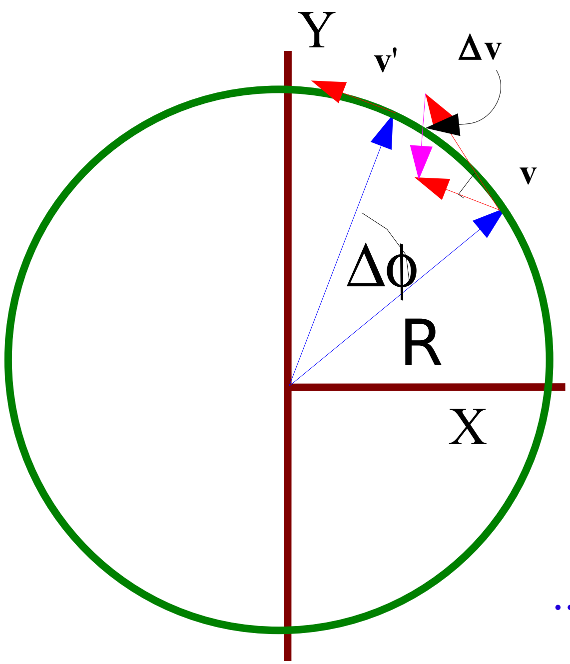
Cinematica

moto circolare non uniforme

l'accelerazione

$$\vec{a}(t)=\frac{d\vec{v}(t)}{dt}=\frac{d}{dt}\rho\,\omega\,\hat{R}_n=\rho\frac{d\omega}{dt}\hat{R}_n+\rho\,\omega\frac{d\hat{R}_n}{dt}=a_T\hat{R}_n-a_r\hat{R}$$

Circolare



poiché

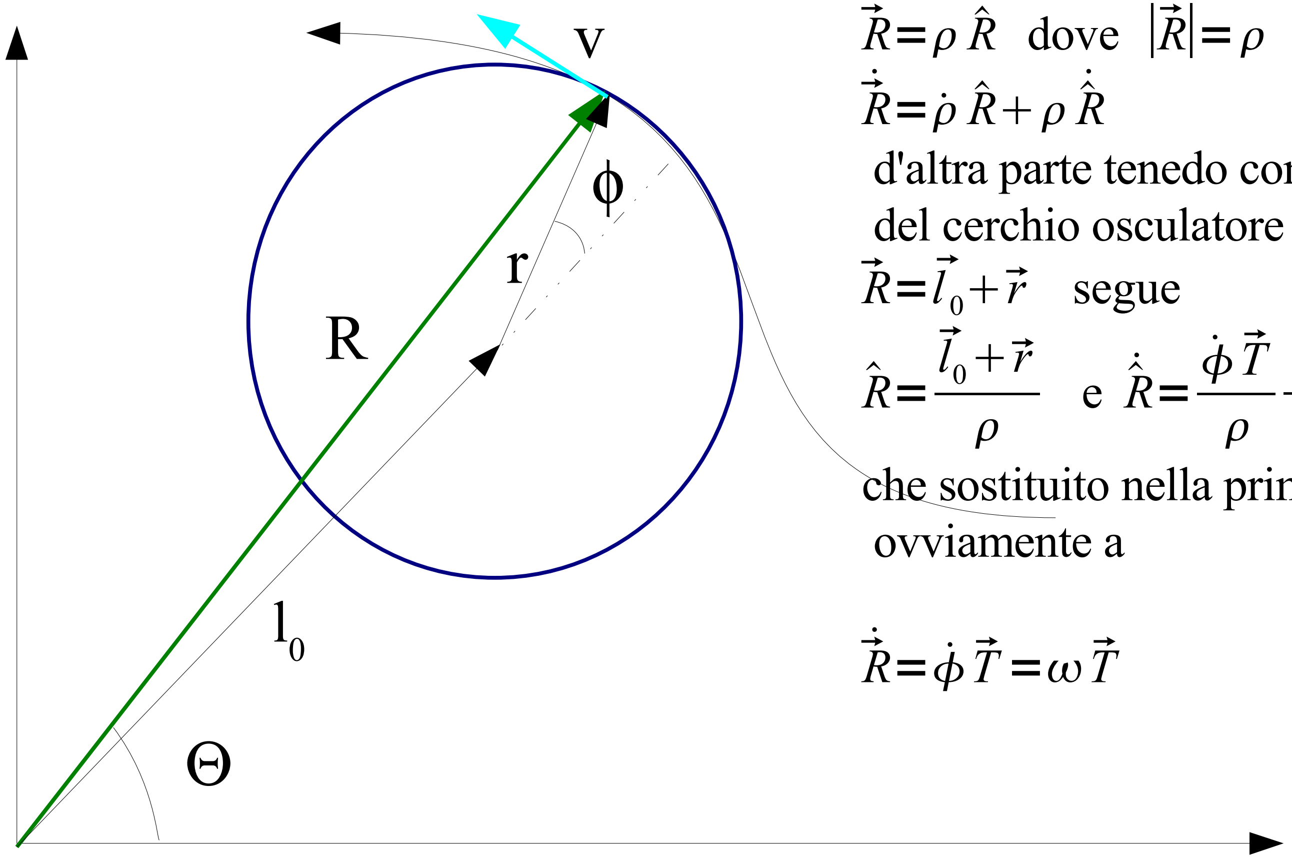
$$\frac{d\hat{R}_n}{dt}=\frac{d\phi}{dt}\frac{d\hat{R}_n}{d\phi}=-\omega\hat{R} \qquad \textit{segue} \qquad a_r=\omega^2\rho$$

Ricorda che $\omega=\frac{d\phi}{dt}$

......qui appaiono le accelerazioni tangenziele e radiali

q

Moto vario su traiettori generica Velocita'



$\vec{R}=\rho\,\hat{R}$ dove $|\vec{R}|=\rho$

$\dot{\vec{R}}=\dot{\rho}\,\hat{R}+\rho\,\dot{\hat{R}}$

d'altra parte tenedo conto del cerchio osculatore

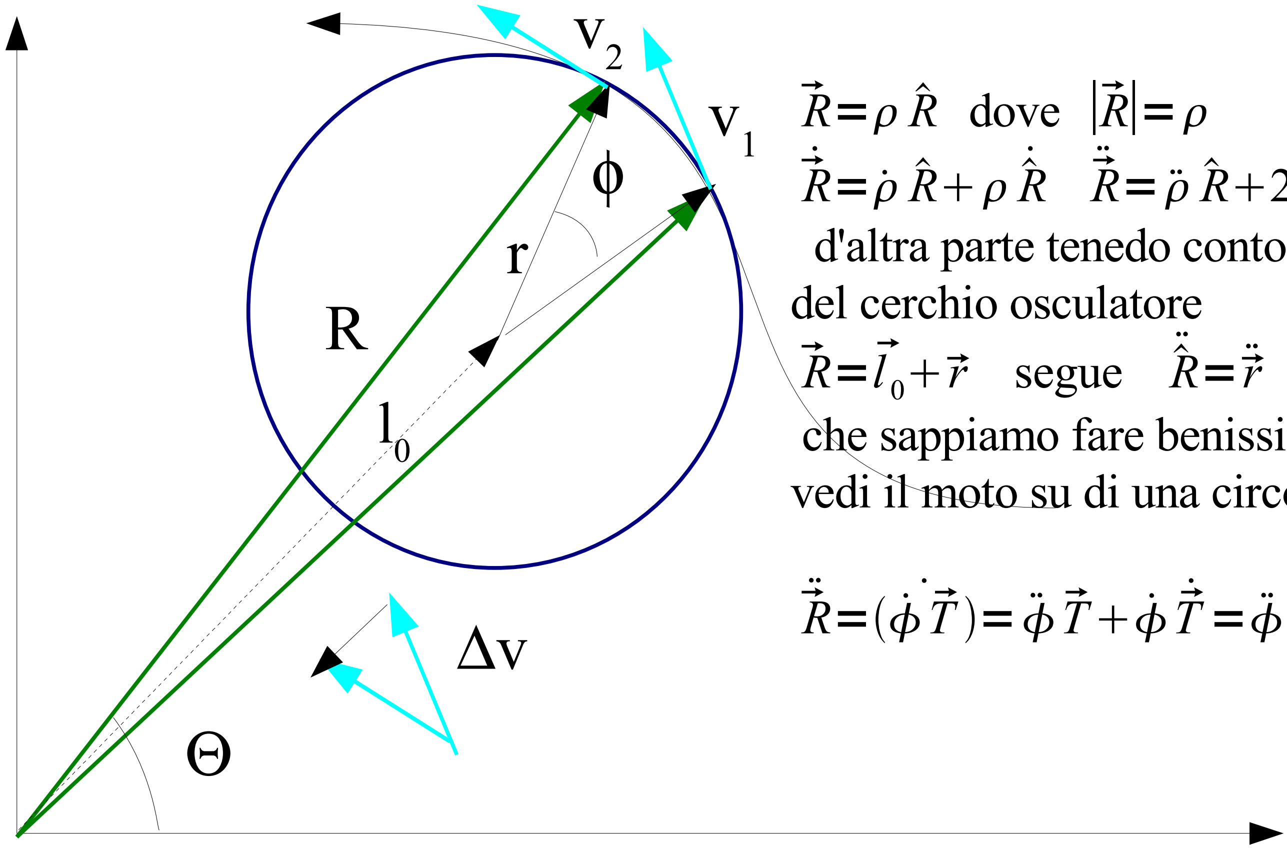
$\vec{R}=\vec{l}_0+\vec{r}$ segue

$\hat{R}=\dfrac{\vec{l}_0+\vec{r}}{\rho}$ e $\dot{\hat{R}}=\dfrac{\dot{\phi}\,\vec{T}}{\rho}-\dfrac{\dot{\rho}\,\hat{R}}{\rho}$

che sostituito nella prima porta ovviamente a

$\dot{\vec{R}}=\dot{\phi}\,\vec{T}=\omega\,\vec{T}$

Moto vario su traiettori generica accelerazione



$\vec{R}=\rho\,\hat{R}$ dove $|\vec{R}|=\rho$

$\dot{\vec{R}}=\dot{\rho}\,\hat{R}+\rho\,\dot{\hat{R}}$ $\quad \ddot{\vec{R}}=\ddot{\rho}\,\hat{R}+2\,\dot{\rho}\,\dot{\hat{R}}+\rho\,\ddot{\hat{R}}$

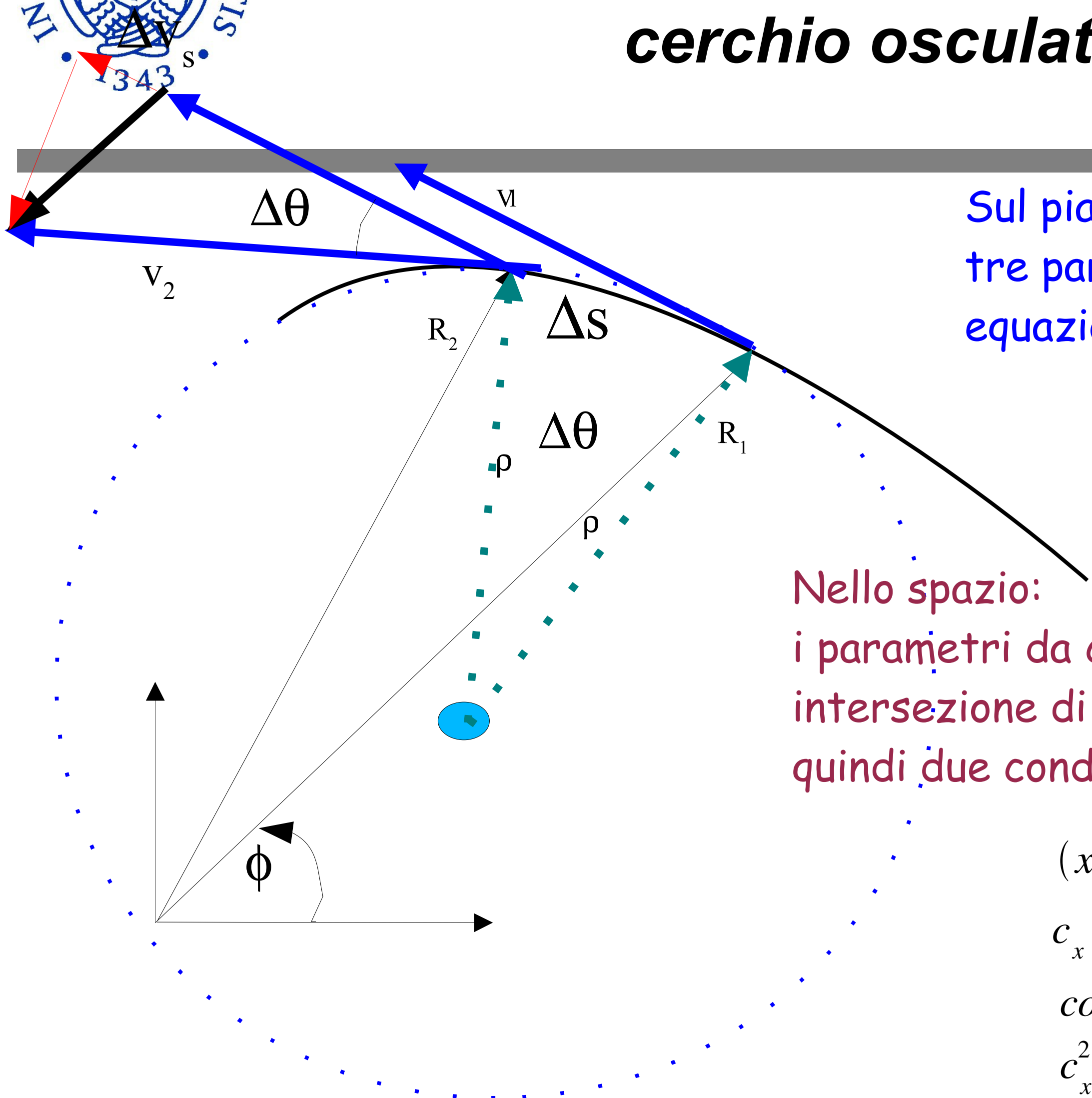
d'altra parte tenedo conto del cerchio osculatore

$\vec{R}=\vec{l}_0+\vec{r}$ segue $\ddot{\hat{R}}=\ddot{\vec{r}}$

che sappiamo fare benissimo vedi il moto su di una circonferenza

$$\ddot{\vec{R}}=(\dot{\phi}\,\vec{T})^{\cdot}=\ddot{\phi}\,\vec{T}+\dot{\phi}\,\dot{\vec{T}}=\ddot{\phi}\,\vec{T}-r\,\dot{\phi}^2\,\hat{r}$$

Cinematica
cerchio osculatore



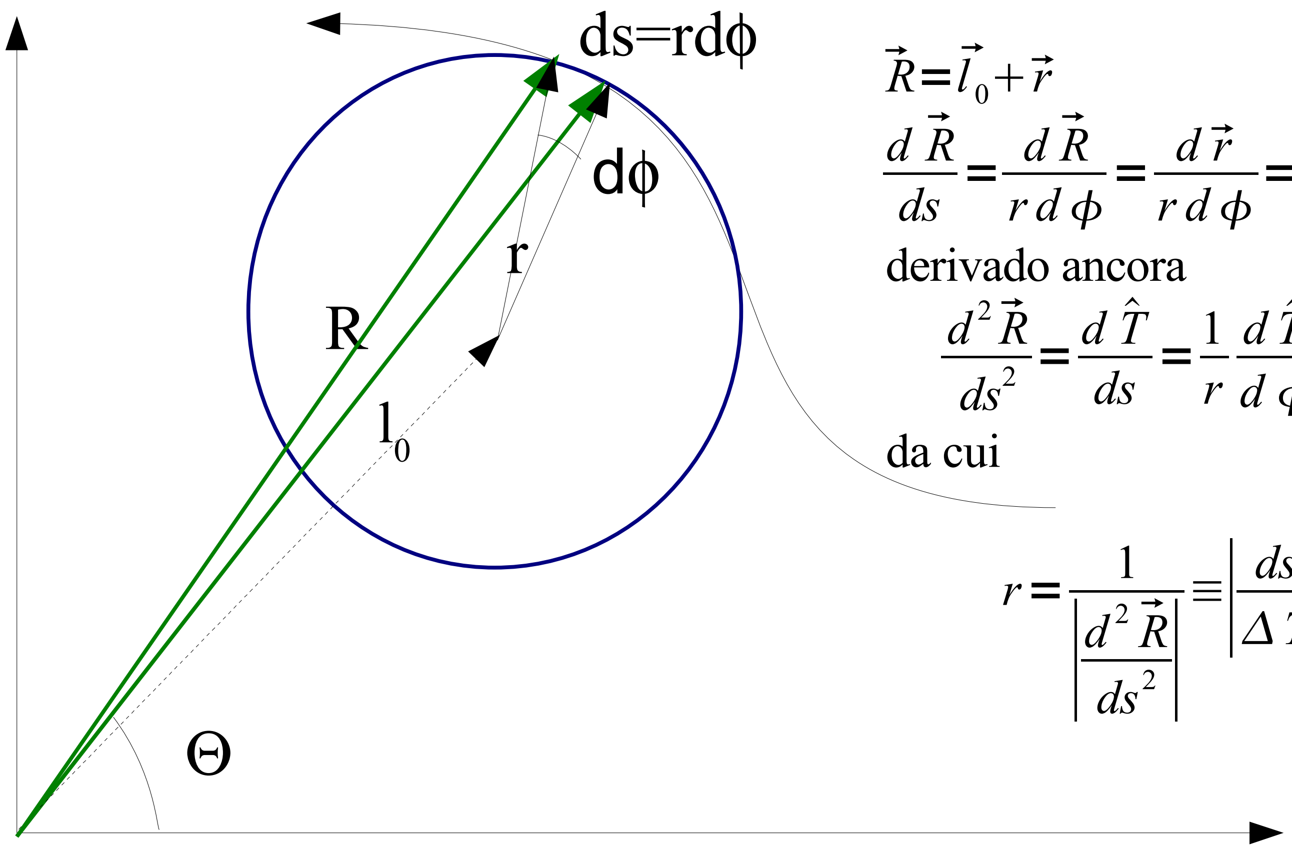
Sul piano :
tre parametri cx,cy,r. una condizione e tre equazioni==>ok

$$(x_1-x_0)^2+(y_1-y_0)^2=\rho$$
$$(x_2-x_0)^2+(y_2-y_0)^2=\rho$$
$$(x_3-x_0)^2+(y_3-y_0)^2=\rho$$

Nello spazio:
i parametri da calcolare sono sei, cx,cy,cz,r,nx,ny
intersezione di un piano con una sfera per tre punti.
quindi due condizioni e tre punti ==> sei equazioni

$$(x_i-x_0)^2+(y_i-y_0)^2+(z_i-z_0)^2=\rho$$
$$c_x x_i+c_y y_i+c_z z_i=d \qquad i=1,2,3$$
$$\textit{con la condizione}$$
$$c_x^2+c_y^2+c_z^2=1$$

Moto vario su traiettori generica
calcolo di r



$$\vec{R}=\vec{l}_0+\vec{r}$$

$$\frac{d\vec{R}}{ds}=\frac{d\vec{R}}{r\,d\phi}=\frac{d\vec{r}}{r\,d\phi}=\frac{d\hat{r}}{d\phi}=\hat{T}$$

derivado ancora

$$\frac{d^2\vec{R}}{ds^2}=\frac{d\hat{T}}{ds}=\frac{1}{r}\frac{d\hat{T}}{d\phi}=\frac{\hat{r}}{r}$$

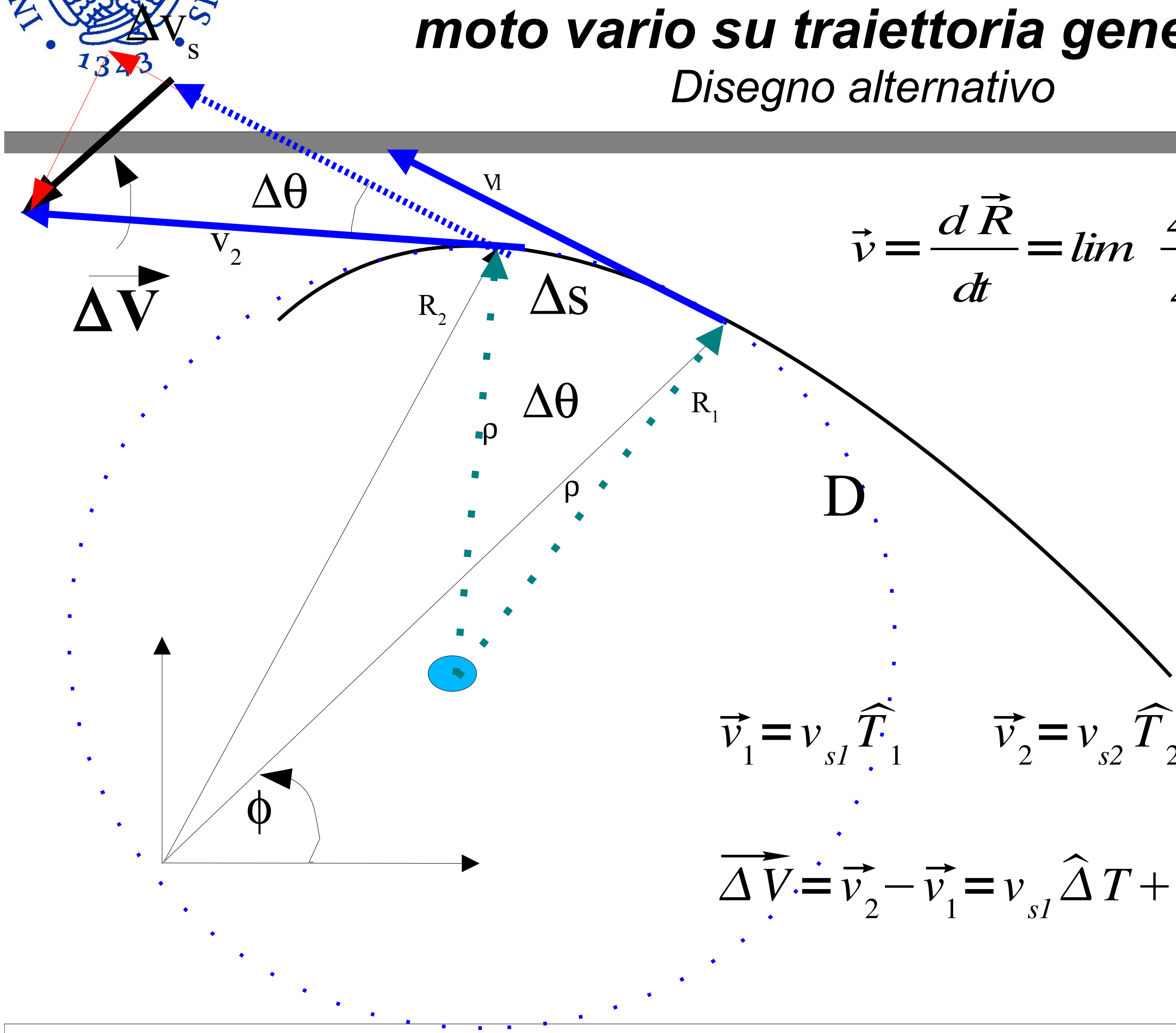
da cui

$$r=\frac{1}{\left|\frac{d^2\vec{R}}{ds^2}\right|}\equiv\left|\frac{ds}{\Delta\hat{T}}\right|$$

Cinematica

moto vario su traiettoria generica

Disegno alternativo



$$\vec{v} = \frac{d\vec{R}}{dt} = \lim \frac{\Delta \vec{s}}{\Delta t} = \frac{ds}{dt}\hat{T} = v_s \hat{T}$$

$$\vec{v}_1 = v_{s1}\hat{T}_1 \qquad \vec{v}_2 = v_{s2}\hat{T}_2 = (v_{s1} + \Delta v_s)(\hat{T}_1 + \widehat{\Delta T})$$

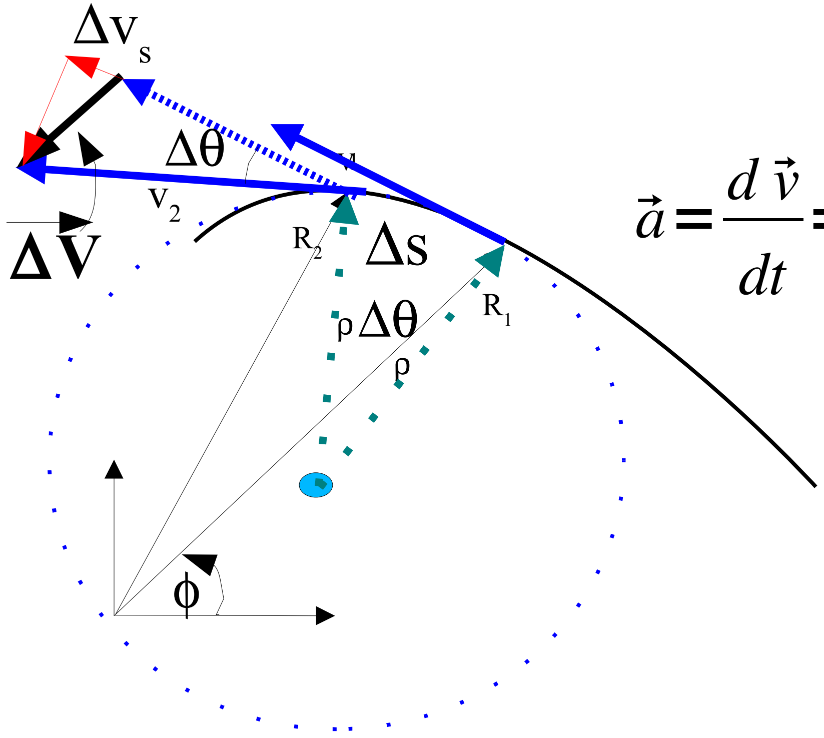
$$\overrightarrow{\Delta V} = \vec{v}_2 - \vec{v}_1 = v_{s1}\hat{\Delta} T + \Delta v_s \hat{T}_1$$

Cinematica

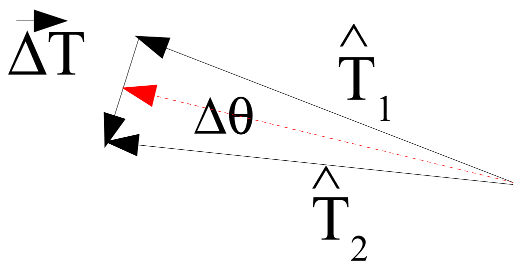
moto vario su traiettoria generica

Disegno alternativo

$$\vec a = lim \frac{\vec v_2 - \vec v_1}{\Delta t} = lim \frac{(v_{s1}+\Delta v_s)(\hat T_1 + \widehat{\Delta T}) - v_{s1}\hat T_1}{\Delta t} = lim \frac{\Delta v_s}{\Delta t}\hat T_1 + v_{s1}\frac{\widehat{\Delta T}}{\Delta t}$$



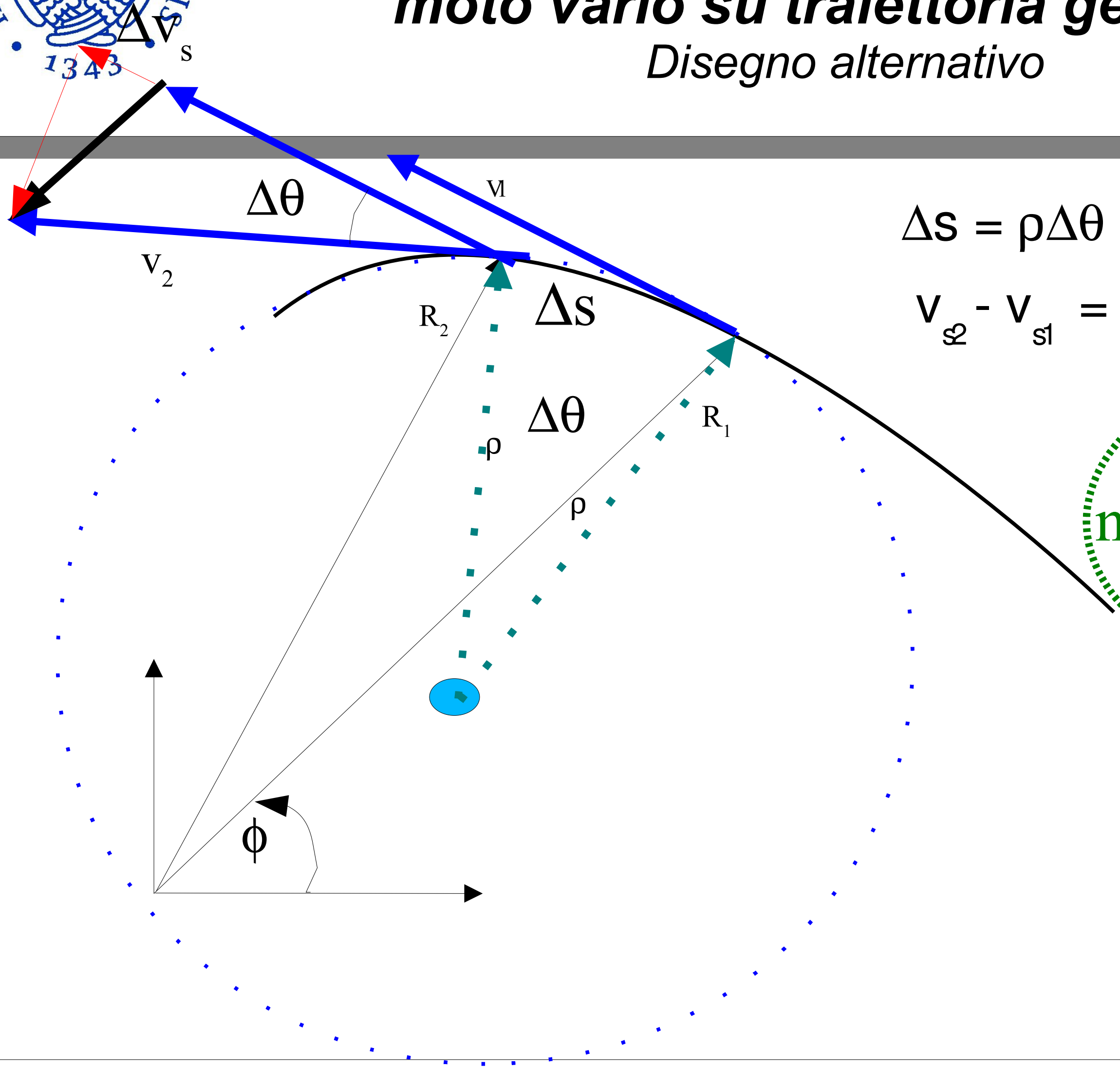
$$\vec a = \frac{d\vec v}{dt} = a_s \hat T + v_s \frac{d\hat T}{dt} = a_s \hat T + v_s \omega \hat T_n$$



Cinematica

moto vario su traiettoria generica

Disegno alternativo



$\Delta s = \rho\Delta\theta$ da cui $v_s = \rho\omega$ e

$v_{s2} - v_{s1} = \Delta v_s = \rho\,\Delta\omega$

nota qui:

$$\omega = \dot{\theta} = \frac{d\theta}{dt}$$

$$\ddot{\theta} = \frac{d^2\theta}{dt^2}$$

$$\omega = \dot{\theta} = \frac{d\theta}{dt}$$

$$\ddot{\theta} = \frac{d^2\theta}{dt^2}$$